Introductory Physics

A First Course In Physics

Zeroth Edition

by
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and Others

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Preface

Thank you for choosing "Introductory Physics: A First Course In Physics." It is the sincere ambition of this project to bring to you a high quality and error free physics textbook suitable for both algebra and calculus based high school physics courses.

While utilizing this textbook it would be of great benefit to the community of users like yourself if you were to report any typographical errors, erroneous historical facts, flawed logic, literary ambiguities, or simply anything that you feel ought to be changed. The feed back from the physics education is of particular interest. If you are a physics educator, we highly encourage you to join the Introductory Physics mailing list or monitor the online forums for errata and other important information.

Charles Edward Pax

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Chapter 1

Measurement

1.1 Fundamental Units

1.1.1 The historic standard of length

Nearly all organized civilizations have at some time employed a unit of length the name of which bore the same significance as does *foot* in English. There can scarcely be any doubt, therefore, that in each country this unit has been derived from the length of the human foot.

Is this accurate?

Where is this information from

But, as might have been expected from such an origin, no two peoples have agreed in the length of their standard. Thus the Greek foot, supposed to represent the length of the foot of Hercules, was 12.14 English inches; the Macedonian foot was 14.08, the Pythian 9.72, and the Sicilian 8.75. In Europe during the Middle Age almost every town had its own characteristic foot; thus in Tome a foot was 11.62 inches, in Milan 13.68, in Brussels 10.86, in Göttingen 11.45, and in Geneva 19.21.

It is probable that in England, after the yard (a unit which is supposed to have represented the length of the arm of king Henry I) became established as a standard, the foot was arbitrarily chosen as one third of this standard yard. The mean length of the male foot in the United States, according to measurements made upon 16,000 men in the United States, is 10.05 inches.

Find the true story and cite it.

What was the name of the study and when was it conducted? Is there a more current source for information?

1.1.2 Relations between different units of length

It has also been true, in general, that in a given country the different units of length in common use, such, for example, as the inch, the hand, the foot, the fathom, the rod, the mile, etc., have been derived either from the lengths of different members of the human body or from equally unrelated magnitudes, and in consequence have been connected with one another by no common multiplier. Thus there are 12 inches in a foot, 3 feet in a yard, 5.5 yards in a rod, 1760 yards in a mile, etc. Furthermore the multipliers are not only different, but are frequently extremely awkward; e.g. there are 16.5 feet, or 5.5 yards, in a rod.

Metric!System
Meter
Metric
Meter
Area
Volume
Area
Are

1.1.3 Relations between units of length, area, volume, and mass

A similar and even worse complexity exists in the relations of the units of length to those of area, capacity, and mass. For example, a square field containing a acre measures 12.649 rods, 69.569 yards, or 208.708 feet on a side; one square rod contains 274.25 square feet; there are 57.75 cubic inches in a quart, and 31.5 gallons in a barrel.

When we turn to the unit of weight we find that the grain, the ounce, the pound, the ton, etc., not only bear different and often very inconvenient relations to one another, but also that none of them bear any simple and logical relations to the units of length. Thus, for example, the pound, instead of being the weight of a cubic inch or a cubic foot of water, or of some other common substance, is the weight of a cylinder of platinum, of inconvenient dimensions, which is preserved in London.

Is this still true?

1.1.4 Origin of the metric system

At the time of the French revolution the extreme inconvenience of existing weights and measures, together with the confusion arising from the use of different standards in different localities, let the National Assembly of France to appoint a commission to devise a more logical system. The result of the labors of this commission was the present metric system, which was introduced in France in 1793, and has since been adopted by the governments of most nations except those of Great Britain and the United States; and even these countries its use in scientific work is practically universal.

Find the real history and cite the source. Does the U.K. use the metric system?

Figure 1.1: Standard meter bar.

fig:meter

How is the meter defined?

1.1.5 The standard meter

The standard length in the *metric* system is called the *meter*. It was originally defined as the distance, at the freezing temperature, between two transverse parallel lines ruled on a bar of platnum (Figure fig:meter 1.1), which is kept in the palace of the Archives in paris.

In order that this standard length might be reproduced if lost, the commission attempted to make it one ten-millionth of the distance from the equator to the north pole, measured on the meridian of Paris. But since later measurements have thrown some doubt upon the exactness of the commission's determination of this distance, we now define the meter, not as any particular fraction of the earth's quadrant, or as the distance between scratches on the above bar, but now derive the unit as _______.

1.1.6 Metric standards of area and volume

Originally, the standard *area* in the metric system was the *are*, which is equal to 100 square meters. Over time, however, it has become customary to express area in terms of the square of a length. While 100 square meters and 1000 square meters can be expressed as 1 and 10 ares respectively, it is perceptually simpler and more common to express them as 100 square meters (m^2) and 1000 m^2 respectively. Liter Mass Gram Liter Gram

The standard unit of volume is called the *liter*[1]. It is the volume of a cube which is one tenth of a meter on a side. Large quantities of volume are sometimes expressed as the product of three lengths, i.e. a pool may be filled with 150 cubic meters (m^3) of water, which is $100 \cdot 150 = 15000$ liters.

1.1.7 The metric standard of mass

In order to establish a connection between the unit of length and the unit of mass, the commission directed a committee of the French Academy to prepare a cylinder of platinum which should have the same weight as a liter of water at its temperature of greatest density, namely, 4 degrees Centigrade, or 39 degrees Fahrenheit. This cylinder was deposited with the standard meter in the Palace of the Archives and now represents the standard of mass in the metric system. It is called the *standard kilogram*. One one-thousandth of this mass and was named the *gram*.

1.1.8 Other metric units

The four standard units of the metric system – the meter, the liter, the gram, and the are – have decimal multiples and submultiples, so that every unit of length, area, volume, or mass is connected with the unit of next higher denomination by an invariable multiplier, and that the simplest possible miltiplier,–namely, ten.

The names of the multiples are obtained by adding to the name of the standard unit the Greek prefices, *deka* (ten), *hecto* (hundred), *kilo* (thousand), and *myria* (ten thousand), while the submultiples are formed by adding the latin prefixes, *deci* (tenth), *centi* (hundredth), and *milli* (thousandth). Examples are listed on table tab:Prefix_Examples 1.1.

Table 1.1: Prefix Usage Examples

1 dekameter	=	10 meters	1 decimeter	=	$\frac{1}{10}$ meter
1 hectometer	=	100 meters	1 centimeter	=	$\frac{1}{100}$ meter
1 kilometer	=	1000 meters	1 millimeter	=	$\frac{1}{1000}$ meter

tab:Prefix_Examples

The most common of these units, with the abbreviations which will henceforth be used for them, are listed on table tab:Unit_Abbrv 1.2. Use the data from SI1998 [1]

1.1.9 Relations between the English and metric units

Table tab:Equivalencies 1.3 gives the relation between the most common English and metric units. Expand an move to an appendix.

Table 1.2: Common Unit Abbreviations

Length	Area	Volume
millimeter (mm) Centimeter (cm) meter (m) kilometer (km)	 square centimeter (cm^2) square meter (m^2)	milliliter (ml) liter (l) cubic meter (m^3)
Mass	Time	Force
gram (g) kilogram (kg)	second (s)	dyne newton (N)

tab:Unit_Abbrv

Table 1.3: English and Metric Equivalencies

1 inch (in.)	=	2.54 cm.	1 cm.	=	0.3937 in.
1 foot (ft.)	=	30.48 cm.	1 m.	=	1.094 yd.
1 mile (mi.)	=	1.609 km.	1 km.	=	0.6214 M.
1 in^2	=	6.45 cm^2	1 cm^2	=	0.1550 in^2
1 ft^3	=	929.03 cm^3	1 m^3	=	1.308 yd^3
1 acre	=	0.405 ha.	1 ha.	=	2.47 acres
1 in^3	=	16.387 cm^3	1 cm^3	=	0.061 in^3
1 ft^3	=	28,317 cm^3	1 m^3	=	1.308 yd^3
1 qt.	=	0.9463 l.	1 l.	=	2.47 acres
1 oz. av.	=	28.35 g.	1 g.	=	0.0353 oz.
1 lb. av.	=	0.4536 kg.	1 kg.	=	2.204 lb.

tab:Equivalencies

This table is inserted chiefly for reference; but the relations 1 in. = 2.54 cm., 1 m. = 39.37 in., 1 kilogram (kg.) = 2.2 lb. should be memorized. On account of its more convenient size, the centimeter, instead of the meter, is frequently used for scientific purposes as the fundamental unit of length. Portions of a centimeter and of an inch scale are shown together in Fig.2.

Time!Second
Mass

1.1.10 The standard unit of time

The *second* is taken among nearly all nations as the standard unit of time. It was originally designated as $\frac{1}{86400}$ part of the time from noon to noon. But since the earth's period of rotation is subject to minute changes, the second is now defined as ______. Find the definition

1.1.11 The three fundamental units

It is evident that measurements of both area and volume may be reduced simply to measurement of length; for an area is expressed as the product of two lengths, and a volume as the product of three lengths. Hence on a single instrument, namely, the meter stick, is all that absolutely essential to the determination of any of these quantities. For these reasons the units of area and volume look upon as *derived* units, depending on one *fundamental* unit, the unit of length.

The weight of an object is determined by the product of its mass by the magnitude of the gravitational force gradient at the object's location. An object's mass can thus be found by weighing it if one knows the magnitude of the force gradient at that point. The *mass* of a body is found by weighing it upon a balance. The operation os something wholly distinct from a measurement of length and requires a new form of instrument. Also the measurement of time is wholly unlike the measurement of either length or mass, and is made with another distinct kind of instrument, namely, a clock, or watch.

Now it is found that just as measurements of area and of volume can be reduced in the ultimate analysis to measurements of length, so the determination of any measurable quantities, such as the pressure in a steam boiler, the velocity of a moving train, the amount of magnetism in a magnet, etc., can be reduced simply to measurement of length, mass, and time. Hence the units of length, mass, and time are considered as *the three fundamental units*, and the three instruments which measure these three quantities, namely, the meter stick, the balance, and the clock, are considered the most fundamental of al instruments.

Whenever any measurement has been reduced to its equivalent in terms of the units of length, mass and time it is dais to be expressed in *absolute* units. Furthermore, since is all scientific work the centimeter, the gram, and the second are now universally recognized as the fundamental units of length, mass, and time reducing all lengths involved to centimeters, all masses to grams, and all times to seconds. The measurement is the often said, for short, to be expressed in C.G.S. (Centimeter-Gram-Second) units. M.K.S?

[1] Replace the leter as the standard volumetric unit.

1.1.12 Questions and problems

1. The Eiffel Tower is 335 m. high. What is its height in feet?

2. A freely falling body, starting from rest, moves 490 cm. during the first second of its fall. Express this distance in feet.

mass/weight?

3. A man weighs 160 lb. What is his weight in kilograms?

4. How many kilograms of butter may be bought for 1 if a pound of butter costs 30 cents?

5. Find the number of millimeters in 5 km. Find the number of inches in 3 mi.

6. Find the number of square rods in a field of 200 ft. on a side. Find the number of square meters in a field 0.3 km. on a side.

7. There are 231 in^3 in a gallon. To what depth d must a tank be made which is 4 yd. long and 4 ft. wide if it is to hold 1500 gal.? What must be the depth of a tank which is to hold 6000 l. if it is 4 m. long and 1.5 m. wide?

Add a small Gnuplot graphic of a box.

1.2 Construction of Standards

1.2.1 Measurement of length

Measuring the length of a body consists simply in comparing its length with that of the standard meter bar kept in paris. In order that this may be done conveniently, millions of rods of the same length as this standard meter var have been made and scattered all over the world. They are our common meter sticks. Theya re divided into 10, 100, or 1000 equal parts, great care being taken to have all the parts of exactly the same length. The method of making a measurement with such a bar is more or less familiar to every one.

1.2.2 Measurement of mass

Similarly, measuring the mass of a body consists in comparing its mass with that of the standard cylinder of platinum, the kilogram of archives. In order that this might be done conveniently, it was first necessary to construct bodies of the same mass as this kilogram and then to make a whole series of bodies whose masses were $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc., of the mass of this kilogram; in other words, to construct a *set of weights*.

1.2.3 Method of duplicating the standard kilogram

To obtain masses exactly equal to the standard kilogram the method of procedure is as follows. The standard cylinder is placed on one pan A of a balance

(Fig. 3),–an instrument which consists essentially of a beam *mn*, supported on a knife edge *C*, and carrying two pans *A* and *B*. Any convenient objects, such as shot, paper, etc., are then added to the pan *B* until the beam balances in the horizontal position, a condition which is indicated by the coincidence of the pointer *P* with the matk *O*. The standard is then removed from *A* and replaced by the body which is desired to make equivalent to it. If the pointer is now found to come back exactly to the mark *O*, the body is considered to have a mass of one kilogram. If the the pointer does not return to *O*, the body is altered (filled away of added to) until coincidence between *P* and *O* is exact.

1.2.4 Method of making a set of masses

To obtain bodies of mass equal to half a kilogram, it is only necessary to take two pieces of metal as nearly alike as possible and file them down together, always keeping them exactly equal to each other, until the balance shows that the two together are exactly equivalent to the standard kilogram. In this way sets of weights may be made which contain any desired masses, e.g. 500 g., 200 g., 100 g., 50 g., 10 g., 1 g., 0.1 g., 0.01 g., 0.001 g., etc.

1.2.5 Method of weighing a body of unknown mass

With the aid of such a set of standard weights, the determination of the mass of any unknown body is made by first placing the body upon the pan *A* and counterpoising with shot, paper, etc., then replacing the unknown body by as many of the standard weights as are required to again bring the pointer back to *O*. The mass of the body is equal to the sum of these standard weights. This is the rigorously correct method of making a weighing, and is called the *method of substitution*.

If a balance is well constructed, however, a weighing may usually be made with sufficient accuracy by simply placing the unknown body upon one pan and finding how many standard weights must then be placed upon the other pan to bring the pointer again to *O*. This is the usual method of weighing. It gives correct results, however, only when the knife edge *C* is exactly midway between the points of support *m* and *n* of the two pans. The method of substitution, on the other hand, is independent of the position of the knife edge.

1.3 Density

1.3.1 Definition of density

When equal volumes of different substances, such as lead, wood, iron, etc., are weighed in the manner described abouve, they are found to have widely different masses. The term "density" is therefore introduced to denote *the mass of unit volume* of a substance.

In the C.G.S. system the cubic centimeter is taken as the unit of volume and the gram as the unit of mass. Hence we say that in this system the density of water is $1\frac{\mathrm{g}}{\mathrm{cm}^3}$, for it will be remembered that the gram was taken as the mass of one cubic centimeter of water. Unless otherwise expressly stated, density is now universally understood to mean density in C.G.S. units, i.e. *the density of a substance is the weight in grams of one cubic centimeter of that substance.* For example, if a block of cast iron 3 cm. wide, 8 cm. long, and 1 cm thick has a mass of 177.6 g., then, since there are 24 cm^3 in the block, the mass of 1 cm^3, i.e. the density, is equal to $\frac{177.6}{24}$ of 7.4.

The density of some of the most common substances is given in table tab:Densities 1.4.

Table 1.4: Densities of Liquids and Solids ($\frac{\mathrm{g}}{\mathrm{cm}^3}$)

tab:Densities

Alcohol	0.79	Hydrochloric acid	1.27
Carbon bisulphide	1.29	Mercury	13.6
Glycerine	1.26	Olive oil	0.91
Aluminum	2.58	Lead	11.3
Brass	8.5	Nickel	8.9
Copper	8.9	Oak	0.8
Cork	0.24	Pine	0.5
Glass	2.6	Platinum	21.5
Gold	19.3	Silver	10.53
Iron (cast)	7.4	Tin	7.29
Iron (wrought)	7.86	Zinc	7.15

etween_mass,_volume,_and_density

1.3.2 Relation between mass, volume, and density

Since the volume of a body is equal to the number of cubic centimeters which it contains, and since its density is by definition the number of grams in one cubic centimeter, its mass, i.e. the total number of grams which it contains, must evidently be equal to its volume times its density. Thus, if the density of iron is 7.4 and if the volume of an iron body is 100 cm^3, the mass of this body in grams must equal $(7.4)(100) = 740$. To express this relation in the form of an equation, let M represent the mass of a body, i.e. its total number of grams; V its volume, i.e. its total number of cubic centimeters; and D its density, i.e. the number of grams in one cubic centimeter; then

$$D = \frac{M}{V}, \text{ or } M = VD, \text{ or } V = \frac{M}{D} \tag{1.1}$$

This equation is merely the algebraic statement of the definition of density.

1.3.3 Distinction between density and specific gravity

The term "specific gravity" is used to denote *the ratio between the weight of a body and the weight of an equal volume of water.* Thus, if a cubic centimeter of iron weighs 7.4 times as much as a cubic centimeter of water, its specific gravity is 7.4. But the density of iron in C.G.S. units is 7.4 $\frac{\text{g}}{\text{cm}^3}$, for by definition density in that system is the mass per cubic centimeter. It is clear, then, that *density in C.G.S. units is numerically the same as specific gravity.*

Specific gravity is the same in all systems, since it simply expresses how many times as heavy a body is an equal volume of water. Density, however, which we have defined as the mass per unit volume, is different in different systems.

Since we shall henceforth use the term "density" to signify exclusively density in C.G.S. system of units, we shall have little further use in this book for the term "specific gravity."[2]

1.3.4 Questions and problems

1. A tank is 8 by 4 by 10.5 cm. What weight of water can it hold?
2. If a rectangular block of wood 5 by 4 by 20 cm. weighs 200 g., what is the density of wood?
3. Find the weight of a liter of mercury. (See table Densities_of_Liquids_and_Solids ??)
4. How many cm^3 in a block of zinc weighing 40 g.?
5. Would you attempt to carry home a block of gold the size of a peck measure? (Consider a peck equal to 8 l.)
6. Find the volume of a block of pine weighing 80 g.
7. The mean density of the earth id 5.53. Its radius is 6370 km. What is its weight in metric tons? (A metric ton is 1000 kilos, about 22oo lb.)
8. Find the volume in liters of a block of platinum weighing 45.5 kilos.
9. Find the density of a steel sphere of radius 1 cm. and weighing 32.7 g.
10. One kilogram of alchol is poured into a cylinder vessel and fills it to a depth of 8 cm. Find the diameter of the cylinder.
11. A capillary glass tube weighs 0.2 g. A thread of mercury 10cm. long is drawn into the tube, when it is found to weigh 0.6 g. Find the diamter of the capillary tube.
12. Find the length of a lead rod 1 cm. in diameter and weighing 1 kg.

[2]Laboratory exercises on length, mass, and density measurements should accompany or follow this chapter. See, for example, Experiments 1, 2, and 3 of the authors' manual.

Chapter 2

Force and Motion

Force_and_Motion

2.1 Definition and Measurement of Force

2.1.1 Distinction between a gram of mass and a gram of force

The SI unit of force is known as the newton (N) and is defined as the force required to accelerate a mass of one kilogram at 1 $\frac{\mathrm{m}}{\mathrm{s}^2}$. If a one kilogram mass is held in the outstretched hand, a downward pull upon the hand is felt. If the mass is 50 kilograms. instead of 1, this pull is so great that the hand cannot be held in place. The cause of this pull is an attractive force which the earth and the mass exert upon each other. In fact, there is a mutual attraction between all the mass of the Universe.

Unfortunately, in common conversation we often fail altogether to distinguish between the concept of mass and the concept of force, and use the same word *gram* to mean sometimes a certain *amount of matter*. That the two ideas are, however, wholly distinct is evident from the consideration that the amount of matter in a body is always the same, no matter where the body is in the universe, while the pull of the earth's surface. It will help to avoid confusion if we reserve the simple term "gram" to denote exclusively "gram of force" wherever we have in mind the pull of the earth upon this mass.

2.1.2 Method of measuring force

When we wish to compare accurately the pulls exerted by the earth upon different masses, we find such sensations as those described in the preceding paragraph very untrustworthy guides. An accurate method , however, of comparing these pulls is that furnished by the stretch produced in a spiral spring. Thus the pull of the earth upon a gram of mass at its surface will stretch a given spring a given distance *ab* (Fig. 4). The pull of the earth upon two grams of mass is found to stretch the spring a larger distance *ac*, upon three grams a still larger distance *ad*, etc. We have only to place a fixed surface behind the pointer and

Magnitude
Direction
Length
Direction

make lines upon it corresponding to the points to which it is stretched by the pull of the earth upon different masses in order to graduate a spring balance (Fig. 5), so that it will thenceforth measure the values of any pulls exerted upon it, no matter how these pulls may arise. Thus if a man stretch the spring so that the pointer is opposite the mark corresponding to the pull of the earth upon two grams of mass, we say that he exerts three grams of force, etc. The spring balance thus becomes an instrument for measuring forces.

2.1.3 The gram of force varies slightly in different localities

With the spring balance it is easy to verify the statement made above that the force of the earth's pull decreases as we reced from the earth's surface; for upon a high mountain the stretch produced by a given mass is indeed found to be slightly less than at the sea level. Furthermore, if the balance is simply carried from point to point over the earth's surface, the stretch is still found to vary slightly. For example, in Chicago it is about one part in 1000 less than it is at Paris, and near the equator it is five parts in 1000 lass than it is near the pole. This is due in part to the earth's rotation, and in part to the fact that the earth is an oblate spheroid, so that in going from the equator toward the pole we are coming closer and closer to the center of the earth. We see, therefore, that the gram of force is not an absolutely invariable unit of force.

2.2 Composition and Resolution of Force

2.2.1 Graphic representation of force

A force is completely defined when its *magnitude* and *direction*, and the *point at which it is applied* are given. Since the three characteristics of a straight line are its *length*, its *direction*, and the *point at which it starts*, it is obviously possible to represent forces by means of straight lines. Thus, if we wish to represent the fact that a force of 8 N, acting in an easterly direction, is applied at the point A (Fig. 6), we draw a line 8 units long, beginning at the point A and extending to the right. The length of this line then represents the magnitude of the force; the direction of the line, the direction of the force; and the starting point of the line, the point at which the force is applied.

Again, if we wish to represent graphically the fact that two forces are acting simultaneously upon a body A (Fig. 7), one being a force of 10 N acting toward the east, and the other a force of 15 N directed toward the north, we have simply to draw two lines from the point A,–one 10 units long and running toward the right, and the other 15 units long running toward the top of the page. These two lines represent completely the two forces in question.

2.2.2 Resultant of two forces acting in the same line

Direction
Equilibrium
Equilibrant

The resultant of two forces is defined as that single force which will produce the same effect upon a body as is produced by the joint action of the two forces.

In general, when a single force acts upon a body which is free to move, the body moves in the direction in which the force acts; but if two oppositely directed forces act simultaneously upon the same body, as when two boys pull in opposite directions on a cart, the effect upon the motion of the cart is just the same as though it were acted upon by a single force equal to the difference between the two forces and acting in the direction of the grater force. For example, if one boy pulls back on the cart with a force of 50 N, while another pulls forward with a force of 75 N, the effect upon its motion is obviously the same as though it were pulled forward with a single force of magnitude 25 N; i.e. *the resultant of two oppositely directed forces applied at the same point is equal to the difference between them, and its direction is that of the greater force.*

if the two forces act in the same direction, the effect upon the motion of the body which they act is that same as though one single force equal in magnitude to the sum of the two forces were acting in their common direction; i.e. *the resultant of two similarly directed forces applied at the same point is equal to the sum of the two forces.*

2.2.3 The resultant of forces acting at an angle

es_acting_at_an_angle

If a body at A is pulled toward the east with a force of 10 N (represented in Fig 8 by the line AC) and toward the north with a force of 10 N (represented in the figure by the line AB), the effect upon the motion of the body must, of course, be the same as though some single force acted somewhere between AC and AB. If the body *moves* under the action of the two equal forces, it may be seen from symmetry that it must move along a line misway between AC and AB, i.e. along the line AR. This line therefore indicates the *direction* of the resultant of the forces AC and AB.

If the two forces are not equal, the the resultant will lie nearer the larger force. As a matter of fact, the experiment of the following graph will show that *if the two given forces are represented in direction and in magnitude by the lines AB and AC (Fig. 9), then their resultant will be exactly represented both in direction and magnitude by the diagonal AR of the parallelogram of which AB and AC are sides.*

2.2.4 Equlibrant

When two or mote forces act upon a body in such a way that no motion results, there is said to be *equilibrium*. Any single force which will preven the motion which one of more forces tends to produce is called an *equilibrant*. Hence the equilibrant of two or more forces is a force equal and opposite to their resultant. Thus if AR (Fig. 9) is the resultant of the forces AB and AC, then AE, taken

Component

equal in length to AR but opposite in direction, is the equilibrant of AB and AC.

Let the rings of two spring balances be hung over the nails B and C in the rail at the top of the blackboard (Fig. 10), and let the weight W be tied near the middle of the string joining the hooks of the two balances. The force of the earth's attraction for the weight W is then exactly equal and opposite to the resultant of the two forces exerted by the spring balances; i.e. OW is the *equilibrant* of the forces exerted by the balances. Let the lines OA and OD be drawn upon the blackboard behind the string, and upon these lines let the distances Oa and Ob be laid off which contain as many units of length as there are units of force indicated by the balances E and F respectively. Then let a parallelogram be constructed upon Oa and Ob as sides. The diagonal of this parallelogram will be found in the first place to be exactly vertical, i.e. in the *direction* of the resultant, since it is exactly opposite of OW; and in the second place the *length* of the diagonal will be found to contain as many units of length as there are units of force in the earth's attraction for W (W must, of course, be expressed in the same units as the balance reading). Therefore the diagonal OR represents in direction, in magnitude, and in point of application the resultant of the two forces represented by Oa and Ob.

In order to test this conclusion more completely, let balances be hung from B and G (Fig. 10). When the parallelogram is constructed as before, its diagonal will be found to have the same length and the same direction as the first. This was to have been expected, since the resultant of Oa and Ob must be in every case equal and opposite to the force of the earth's attraction upon W.

2.2.5 Component of a force

Whenever a force acts upon a body in some other direction than that in which the body is free to move. it is clear that the full effect of the force cannot be spent in producing motion. For example, suppose that a force is applied in the direction OR (Fig. 11) to a car on an elevated track. Evidently OR produces two distinct effects upon the cat: on the one hand it moves the car along the track, and on the other it presses down against the rails. These two effects might be produced just as well by two separate forces acting in the directions OA and OB respectively. The value of the single force which, acting in the direction OA, will produce the same motion of the car on the track as is produced by OR, is called the *component* of OR in the direction OA. Similarly the value of the single force which, acting in the direction OB, will produce the same pressure against the rails as produced by the force OR, is called the component of OR in the direction OB. In a word, *the component of a force in a given direction is the effective value of the force in that direction.*

2.2.6 Magnitude of the component of a force in a given direction

Since, from the definition of component just given, the two forces, one to be applied in the direction OA and the other in the direction OB, are together to be exactly equivalent to OR in their effect on the car, their magnitudes must be represented by the sides of a parallelogram of which OR is the diagonal. For in section 2.2.3 it was shown that if any one force is to have the same effect upon a body as two forces. Hence conversely, if two forces are to be equivalent in their joint effect to a single force, they must be sides of the parallelogram of which the single force is the diagonal. Hence the following rule: *To find the component of a force in any given direction, construct upon the given force as a diagonal a rectangle the sides of which are respectively parallel and perpendicular to the direction of the required component. The length of the side which is parallel to the given direction represents the magnitude of the component which is sought.* Thus, in the above illustration, the line Om completely represents the component of OR in the direction of OA, and the line On represents the component of OR in the direction of OB.

It will be seen from Fig. 11 that as OR becomes more and more nearly parallel to the track, the component of OR along the track becomes larger and larger, while the component perpendicular to the track becomes smaller and smaller. When OR is parallel to the track, the component at right angles to the track becomes zero. When OR is perpendicular to the track, its component parallel to the track becomes zero.

2.2.7 Component of weight which is parallel to an inclined plane

To apply the test of experiment to the conclusions of the preceding paragraph, let a wagon be placed upon an inclined plane (Fig. 12), the height of which, BC, is equal to one half its length AB. In this case t he force acting on the wagon is the weight of the wagon, and its direction is downward. let this force be represented by the line OR. Then by the construction of the preceding paragraph, the line Om will represent the value of the force which is pulling the carriage down the plane, and the line On the value of the force which is producing pressure against the plane. Now since the triangle ROm is similar to the triangle abc, their sides being mutually perpendicular, we have

$$\frac{Om}{OR} = \frac{bc}{ab}$$

i.e. in this case, since bc is equal to one half of ab, Om is one half of OR. Therefore the force which is necessary to prevent the wagon from slaiding down the plane should be equal to one half its weight. To theat this conclusion, let the wagon be weighed on the spring balance and then placed on the plane in the manner shown in the figure. The pull indicated by the balance will, indeed, be found to be one half of the weight of the wagon.

The equation $\frac{Om}{OR} = \frac{bc}{ab}$ shows that in general *the force which must be applied to a body to hold it in place upon an inclined plane bears the same ratio to the weight of the body that the height of the plane bears to its length.*

2.2.8 Component of gravity effective in producing the motion of a pendulum

When a pendulum is drawn aside from its position of rest (Fig. 13), the force acting on the bob is its weight, and the direction of this force is vertical. Let it be represented by the line OR. The component of this force in the direction in which the bob is free to move is On, and the component at right angles to this direction is Om. The second component Om simply produces stretch in the string and pressure upon the point of suspension. The first component On is alone responsible for the motion of the bob. A consideration of the figure shows that this component becomes larger and larger the greater the displacement of the bob. When the bob is already beneath the point of support the component producing motion is zero. Hence the pendulum can permanently at rest only when its bob is directly beneath the point of suspension.[1]

2.2.9 Questions and problems

1. Represent graphically a force of 20 N acting southeast and a force of 25 N acting southwest at the same point. What will be the magnitude of the resultant, and what will be its approximate direction?

2. The engines of a steamer can drive it 19 km an hour. How fast can it go up stream in which the current is 2 $\frac{\mathrm{m}}{\mathrm{s}}$? How fast can is come down stream?

3. The wind drives a steamer east with a force which would carry it 19 km per hour, and its propeller is driving it south with a force which would carry it 24 km per hour. what distance will it actually travel in an hour? Draw a diagram to represent the exact path.

4. A boy pulls a loaded sled weighing 890 N up a hill which rises 1 m in 5. Neglecting friction, how much force must he exert?

5. What force will be required to support a 75 N. ball on an inclined plane of which the length is 10 times the height?

6. A boy is able to exert a force of 100 N. How long an inclined plane must he have in order to push a truck weighing 425 N up to a doorway 1 m above the ground?

2.3 Gravitation

2.3.1 Newton's law of universal gravitation

In order to account for the fact that the earth pulls bodies toward itself, and at the same time to account for the facts that the moon and the planets are held in

[1] It is recommented that the formal study of the laws of the pendulum be reserved for laboratory work (see Experiment 17, authors' manual).

their respective rorbits around the earth and sun, Sir Isaac newton (1642-1727) first announced the law which is now known as the law of universal gravitation. This law asserts first that *any two bodies in the universe attracts every other body with a force which varies inversely as the square of the distance between them.* This means that if the distance between the two bodies considered is doubled, the force will become only one fourth as great; if the distance is made three, four, or five times as great, the force will be reduced to one ninth, on sixteenth, or one twenty-fifth of its original value, etc. This law of mutual attraction may be expressed using the formula

$$F = \frac{\mathrm{G} m_1 m_2}{d^2} \tag{2.1}$$

where F is the force of attraction between mass m_1 and mass m_2, and d is the distance between them. G is called the gravitational constant and $\mathrm{G} = 6.67 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$.

2.3.2 Gravitational force varies with distance

If a body is spherical in shape and of uniform density, it attracts external bodies with the same force as though its mass were concentrated at its center. Since, therefore, the distance from the surface to the center of the earth is about 64360 kilometers, we learn from newton's law that a body 64360 kilometers above the earth's surface would weigh one forth as much as it does at the surface. It will be seen, then, that if a body be raised but a few meters or even a few kilometers above the earth's surface, the decreases in its weight must must be a very small quantity, for the reason that a few meters or a few kilometers is a small distance compared with 64360 kilometers. As a matter of fact, a 1kg mass which would weigh 9.801 N. at sea level would weigh about 9.781 N. at the top of a mountain 6.436 kilometers high.

2.3.3 Center of gravity

From the law of universal gravitation it follows that every particle of a body upon the earth's surface is pulled toward the earth. It is evident that the sum of all these pulls on the particles of which the body is composed must be equal to the total pull of the earth upon the body, i.e. to the weight of the body. Now it is always possible to find on a single point in a body at which a single force equal in magnitude to the weight of the body and directed upward can be applied so that the body will remain at rest in whatever position it is placed. This point is called the *center of gravity* of the body. Since this force counteracts entirely the weight of the body, it must be equal and opposite of the resultant of all the small forces which gravity is exerting upon the different particles of the body. Hence the center of gravity may be defined as the point of application of the resultant of all the little downward forces; i.e. it is *the point at which the entire weight of the body may be considered as concentrated.* The earth's attraction for a body is therefore always considered not as a multitude of little forces but as one single force F (Fig. 14) equal to the weight of the body and applied at its center of gravity G.

2.3.4 Method of finding center of gravity experimentally

From the above definition it will be seen that the most direct way of finding the center of gravity of any flat body, like that shown in Fig. 15, is to find the point upon which it will balance.

To illustrate another method for finding the center of gravity of the zinc, let it be supported from a pin stuck through a hole near its edge, e.g. b (Fig. 15). Let a plumb line be hung from the pin, and let a line bn be drawn through b on the surface of the zinc, parallel to and directly behind the plumb line. Let the zinc be hung from another point a, and another line am drawn in similar way.

The point of intersection of the two lines is at the center of gravity. For since the earth's attraction may be considered as a single force applied at the center of gravity, the zinc can remain at rest only when the center of gravity is directly beneath the oint of support. It must, therefore, lie somewhere on the line am. For the same reason it must lie on the line bn. But the only point which lies on both of these lines is their point of intersection G.

2.3.5 Stable equilibrium

A body is said to be in *stable equilibrium* if it tends to return to its origional position when given a slight displacement. A endulum, a chair, a sube resting on its side, a cone resting on its base, are illustrations.

In general, a body is in stable equilibrium whenever a slight displacement thends to raise its center of gravity. Thus, in Fig. 16 all of the bodies A, B, C, D are in stable equilibrium, for in oorder to overturn any one of them, its center of gravity G must be raised through the height ai. If the weights are all alike, that one will be most stable for which ai is greatest.

The condition of stable equiligrium for bodies which rest upon a horizontal plane is that a vertical line through the center of gravity shall fall within the base, the base being defined as the polygon formed by connecting the point at which the body touches the plane, as ABC (Fig. 17); for its is clear that in such a case a slight displacement must raise the center of gravity along the arc of which OG is the radius. If the vertical line drawn through the denter of gravity fall outsice the base, as in Fig. 18, the body must always fall.

The condition of stable equilibrium for bodies supported from a single point is that the point of support be above the center of gravity. For example, the beam of a balance cannot be in stable equilibrium so that it will return to the horizontal position when slightly displaced, unless its center of gravity g (Fig. 3, p.7) is below the knife edge C.

2.3.6 Neutral equilibrium

A body is said to be in neutral equilibrium when, after a slight displacement, it tends neither to return to its original position nor to move farther from it. Examples of neutral equilibrium are a spherical ball lying on a smooth plane, a cone lying on its side, a wheel free to rotate about a fixed axis through its center, or any body is in neutral equilibrium when a slight displacement neither raises nor lowers its center of gravity.

2.3.7 Unstable equilibrium

A body is in unstable equilibrium when after a slight displacement it tends to move farther from its original position. A cone balanced on its point or an egg on its end are examples. In all such cases a slight displacement always lowers the center of gravity and the motion when continues until the center of gravity is as low as circumstances will permit. The condition for unstable equilibrium in the cases of a body supported by a point is that the center of gravity shall be above the point of support. Fig 19 illustrates the three kinds of equilibrium.

2.3.8 Questions and problems

1. A body weighs 981 N. at the earth's surface. What will it weigh 4000 km. above the surface? What will it weigh 1000 km. above the surface? (Take the earth's radius as 6436 km.)

2. What is the object of ballast in a ship?

3. Explain why the toy shown in Fig. 20 will not lie upon its side, but instead rises to the vertical position. Does the center of gravity actually rise?

4. If a lead pencil is balanced on its point on the finger it will be in unstable equilibrium, but if two knives are stuck into it, as in stable equilibrium. Why?

5. Why does a man lean forward when he climbs a hill?

2.4 Uniformly Accelerated Motion

2.4.1 Uniform motion

If while a body moves across a distance during a period of time if the ratio of distance moved to the time passed during that move is equal for all moments in time during the body's motion, its motion is said to be *uniform*. Thus the motion of a train moving between stations can be considered as generally uniform if one does not consider the periods during which the train's motion

Velocity

changes during its departure of one station and its arrival at another. The motion of a body is normally given as a function of time t, $f(t)$.

$$f(t) = f_x(t) + f_y(t) + f_z(t)$$

For now we will concern ourselves with motion along a single axis, namely the x axis, our function will be $f_x(t)$.

2.4.2 Velocity

When the motion of a body is uniform, its *velocity* is defined as the distance which it traverses per unit of time. This is called *constant velocity.* When the motion of a body is not uniform, its velocity at any instant is defined as the distance which it would travel in a given period of time if at that instant its motion were to become uniform, also called *instantaneous velocity.* Sometimes we are only concerned with the distance a body travels and the period of time that travel took. This is called the *average velocity* $v_{\text{avg}}$and is found by taking the difference between the initial velocity $\vec{v}_i$ and the final velocity $\vec{v}_f$ over the period t

$$v_{avg} = \frac{\vec{v}_i - \vec{v}_f}{t} \tag{2.2}$$

2.4.3 Acceleration

If a train from rest has a velocity of one meters per second at the end of the first second, a velocity of two meters per second at the end of the second second, of three meters per second at the end of the third second, etc., its motion is said to be *uniformly accelerated.* The *gain* in the *velocity* of such a body *per second* is called its *acceleration*; e.g. in the case above, the acceleration is one meter per second per second or $1\frac{\mathrm{m}}{\mathrm{s}^2}$. In general, then, acceleration is defined as *the rate at which velocity changes.* If the motion is uniformly acceleration, its acceleration is equal to the velocity gained per second.

2.4.4 Relative distances traversed by a falling body is one, two, three, four, etc., seconds

_two,_three,_four,_etc.,_seconds

The simplest case of uniformly accelerated motion is that of a falling body. Since, however, a freely falling body acquires velocity so rapidly that it is difficult to make observations upon it directly, Galileo hit upon the plan of studying the laws of falling bodies by observing the motion of a ball rolling down an inclined plane. He found that a body falls exactly 4 times as far in 2 seconds as in 1, 9 times as far in 3 seconds, 16 times as far in 4 seconds, 25 times as far in 5 seconds, etc.

To test the correctness of these results, let a grooved board about 16 ft. long be supported as in Fig. 22, one end being about a foot and a half above the other. Let supports be introduced near the middle, if necessary, so that the plane will not sag. Let the metronome, or clock beating seconds, be started, and the marble A released at the instant of one click of the metronome. Let the block B be placed at the such a distance down the incline that the click produced by the impact of the ball upon it coincides exactly with, for example, the fourth click of the metronome. The time of fall is then three seconds. Let the distance traversed be measured. Then let B be placed at a distance equal to $\frac{4}{9}$ of this distance and the experiment repeated. The ball will strike B exactly at the end of two seconds. At a distance equal to $\frac{1}{9}$ of the first distance the impact will occur at the end of one second, etc. An interesting variation of this experiment is to have three grooves, three marbles, and three blocks B set at distances 1, 4, and 9 from the common starting point. If the marbles are all released at the instant of one click, a marble will strike a block at the exact instant of each of the three succeeding clicks.

Figure 2.1 contains a graph of two values. The x axis represents time while the y axis on the left side represents the velocity of the moving body during the period in question and the right side represents the distance traveled. From the graph we can see that the body is accelerating because its velocity is increasing by $9.81\frac{\text{m}}{\text{s}}$ each second. We can also see that from the distance graph that the distance traveled during each second increases by 9.81m from the second before it.

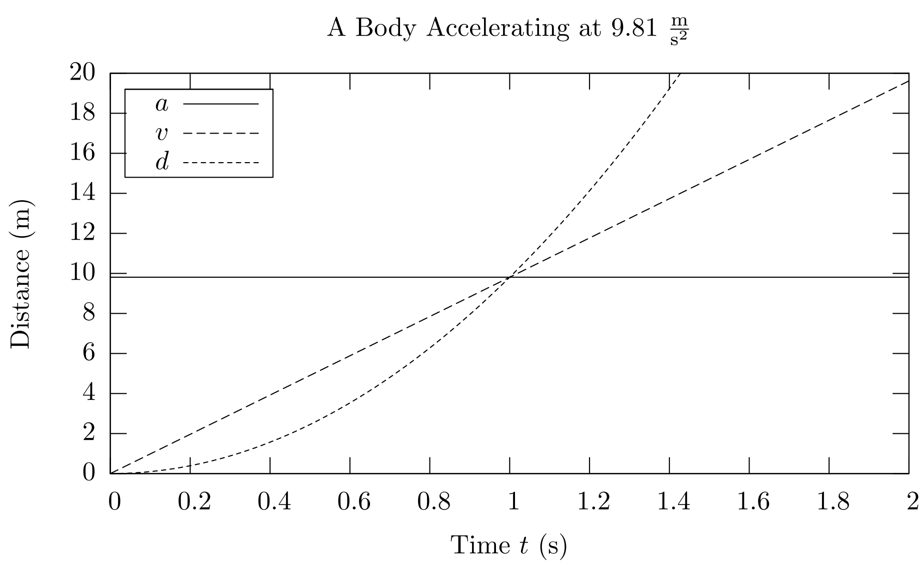

Figure 2.1: A Body Accelerating at $9.81\frac{\text{m}}{\text{s}^2}$ fig:FallingBody

2.4.5 Velocity acquired per second by the marble

cquired_per_second_by_the_marble

In the last paragraph we investigated the distances traversed in one, two, three, etc., seconds. Let us now investigate the *velocities* acquired on the same inclined plane in one, two, three, etc., seconds.

Let a second grooved board M be placed at the bottom of the incline, in the manner shown in Fig. 22. To eliminate friction it should be given a slight slant, just sufficient to cause the ball to roll along it with uniform velocity. Let the ball be started at a distance D up the incline, D being the distance which it was found in the last experiment to roll during the first second. It will then just reach the bottom of the incline at the instant of the second click. Here it will be freed from the accelerating force of gravity, and will therefore more along the lower board with the velocity which it had at the end of the first second. It will be found that when the block is placed at a distance exactly equal to $2D$. If the ball is started at a distance $4D$ up the incline, it will take it two seconds to reach the bottom, and it will roll a distance $4D$ in the next second; i.e. in two seconds it acquires a velocity $4D$. In three seconds it will be found to acquire a velocity $6D$, etc.

The experiment shows, first, that the increase in velocity each second is the same, namely $2D$, and that the motion is therefore uniformly accelerated. Furthermore, it shows that *uniformly accelerated motion the acceleration (velocity gained per second) is measured by twice the distance passed over in the first second.*

2.4.6 Distances traversed during successive seconds

If we subtract from the distance traversed in two seconds the distance traversed in one second, we get $4D - D = 3D$, which is the distance traversed during the second second. Similarly, if subtract the distance traversed in two seconds from the distance traversed in three seconds, we obtain $9D - 4D = 5D$, which is the distance traversed turing the third second. In the same way the distance traversed in the fourth second is $7D$, etc.

2.4.7 Tabular statement of the laws of falling bodies

nt_of_the_laws_of_falling_bodies

Putting together the results of the last three paragraphs, we obtain the following table, in which D represents the distance fallen the first second.

Velocity_acquired_per_second_by_the_marble

Since D was shown in section 2.4.5 to be equal to one half the velocity acquired per second, i.e. one half the acceleration a, we have at once, by substituting $\frac{1}{2}a$ for D in the last row of the table,

$$v = at \tag{2.3}$$ v=at

$$s = \frac{1}{2}a2(2t-1) \tag{2.4}$$ s=1/2a2(2t-1)

$$S = \frac{1}{2}at^2 \tag{2.5}$$ S=1/2at^2

Table 2.1: Falling Body Data

Number of Seconds	Velocities at the End of Each Second	Spaces Fallen Each Second	Total Distance Fallen
1	$2D$	$1D$	$1D$
2	$4D$	$3D$	$4D$
3	$6D$	$5D$	$9D$
4	$8D$	$7D$	$16D$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	$2tD$	$(2t-1)D$	t^2D

Falling_Body_Data

These formulas are simply the algebraic expression of the facts brought out by our experiment; but the reasons for these facts may be seen as follows.

Since in uniformly accelerated motion the acceleration a is the velocity gained per second, it follows at the once that the velocity v gained in t seconds is $v = at$. This is bormual 2.3 above.

To obtain formula 2.5 we have only to consider that the total distance S traversed by any moving body is t seconds is the *average* velocity multiplied by t, the number of seconds. But the average in uniformly accelerated motion is the *mean* of the initial and final velocities. Hence, if the body from rest and acquires in t seconds a velocity v, its average velocity is $\frac{0+v}{2} = \frac{v}{2}$. Hence the space traversed is given by $S = \frac{v}{2}t$. By substituting in this equation $v = at$ we get, $S = \frac{1}{2}at^2$. To obtain 2.4 we have only to subtract from the space traversed in t seconds that traversed in $(t-1)$ seconds. Thus $s = \frac{1}{2}at^2 - \frac{1}{2}a(t-1)^2 = \frac{1}{2}a(2t-1)$.

To illustrate the use of these results, suppose that a body rolling down an inclined plane is known to move over 10 cm. the first second, and that we are required to find 2.3 what velocity it will have at the end of the tenth second, 2.4 how far it will roll during the tenth second, and 2.5 how far it will have rolled during the 10 seconds.

Since the acceleration is $2 \cdot 10 = 20$, the answers are firstly $v = at = 20 \cdot 10 = 200\frac{\mathrm{m}}{s}$ secondly $s = \frac{1}{2}a(2t-1) = \frac{1}{2}20(20-1) = 190$ m and thirdly $S = \frac{1}{2}at^2 \cdot 20 \cdot 100 = 1000$ m.

2.4.8 Acceleration of a freely falling body

If in the above experiment the slope of the plane be made steeper, the results will be precisely the same, except that the acceleration has a larger value. If the board is tilted until it becomes vertical, the body becomes a freely falling body. In this case the distance traversed the first second is found to be 490 cm., or 16.08 ft. Hence the acceleration expressed in centimeters is 980, in feet 32.16. This acceleration of free fall, called the *acceleration of gravity*, is usually denoted by the letter g.

To illustrate the use of this constant, suppose we wish to know how far a body will fall in 10 seconds. We have

$$S = \frac{1}{2}gt^2 = \frac{1}{2} \cdot 980 \cdot 100 = 49,000 \text{ m.} = 490 \text{ m.}$$

2.4.9 Rates of fall of different bodies

It is a fact of familiar observation that very light bodies, such as feathers and bits of paper, fall bery much more slowly than pieces of wood of iron. Previous to Galileo's time it was taught in the schools that heavy bodies fall toward the earth with " velocities proportional to their weights." Galileo demonstrated thte incorrectness of this view by his famous experiments conducted from the leaning tower of Pisa (Fig. 23. In the presence of the professors and students of the University of Pisa he dropped ball s of different sizes and materials from the top of the tower, 54.9 m high, and showed that they fell in pratically the same time. He showed that even very light bodies, like paper, fell with velocities which approached more and more nearly those of heavy bodies the more compactly they were wadded together. He inferred from these experiments that all bodies, even the lightest, would fall at the same rate were it not for the resistance offered by the air,–an inference which could not be verified at that time because the air pump had not yet been inventer. After its invention, sixty years later, by Otto von Guericke, Galileo's inference was verified in the following way. A feather and a coin were placed in a glass tube four or five feet long, and the air pumped out. When the tube was then inverted the coin and the feather fell side by side from the top ot the bottom (Fig. 24).

2.4.10 Velocity acquired in falling from a given height

d_in_falling_from_a_given_height

If we wish to find with what velocity a body which falls from a given height S, say 200 m, will strike the earth, we can first get the time of descent from equation 2.5, on page 22, and then get the velocity from equation 2.3, page 22. Thus from equation 2.5,

$$t^2 = \frac{2 \cdot 200}{9.8}, \text{ or } t = \sqrt{\frac{2 \cdot 200}{9.8}},$$

and from equation 2.3,

$$v = 9.8t = 9.8\sqrt{\frac{2 \cdot 200}{9.8}} = \sqrt{2 \cdot 9.8 \cdot 20000} = 62.6 \text{ m}$$

If we write the symbols instead of the numbers, we see that the formual connection v and S is

$$v = \sqrt{2gS} \tag{2.6}$$

v=sqrt(2gS)

2.4.11 Height to which a body projected vertically upward will rise

Since the earth emparts to a freely falling body a acceleration of 9.8 $\frac{\mathrm{m}}{\mathrm{s}^2}$, a body shot vertically upward must lose 9.8 m of velocity during each second of its ascent. Hence the number of seconds during which it is rising is obtained by dividing its initial velocity by 9.8. The *time* of ascent t is therefore given in terms of g and the initial velocity by $t = \frac{v}{g}$.

To find the *height* of ascent, we have only to consider that the total distance traveled is that *average* velocity times the time. But since the hody starts with a velocity v and reaches the top of its path with a velocity 0, the average velocity during the ascent must be $\frac{v+0}{2} = \frac{v}{2}$. Hence the distance S through which it tises is given by

$$S = \frac{v}{2}t.$$

If we substitute in this value of t given above, namely $\frac{v}{g}$, we get

$$S = \frac{v^2}{2g}, \text{ or } v = \sqrt{2gS} \tag{2.7}$$

S=v^2/2g

Thus, if a body is shot upward with a velocity of $10\frac{\mathrm{m}}{s}$, equation 2.7 tells us that it will rise to a height of $\frac{10^2}{9.8s} = 5.1$ m. Or if a body is seen to rise to a height of 500 m., we know from the second form of 2.7 that it must have been shot upward with a velocity of $v = \sqrt{2 \cdot 9.8 \cdot 50000} = 93.6\frac{\mathrm{m}}{s}$.

We learn also from equation 2.7 that the velocity with which a body must be projected upward to rise to a given height is the same as the velocity whish it will acquire in falling from the same height (see equation 2.6, page 24).

2.4.12 Questions and problems

Go back and add these questions

2.5 Newton's laws of Motion

2.5.1 First law–Inertia

In 1686 Sir Isaac Newton formulated three statements which embody the results of universal observation and experiment on the relations which exist between force and motion. The statement of the first law is: *Every body continues in its state of rest or uniform motion in a straight line unless impelled by external force to change that state*. This statement is based upon such familiar observations as the following. Bodies on a moving train tend to move toward the forward end when the train s stops moving, and toward the rear end when the train starts moving; i.e. they tend in each case to continue in their previous state whether that were one of rest of motion. That a moving body also tends to move on in a *straight line in the direction of its motion* is seen from such facts as that mud

Inertia
Momentum

flies off tangentially from a rotating carriage wheel, or water from a whirling grindstone. This property which all matter possesses of resisting any attempt to start it if at rest, to stop it if in motion, or in any way to change either the direction or amount of its motion, is called *inertia*.

2.5.2 Centrifugal force

It is inertia alone which prevents the planets from falling into the sun; which causes a rotating sling to pull on the hand until the stone is released, and which then causes the stone to fly off tangentially. It is inertia which makes rotating liquids move out as far as possible from the axis of rotation (Fig. 25); which makes fly wheels sometimes burst; which makes the equatorial diameter of the earth greater than the polar; which makes the heavier milk move out farther than the lighter cream in the dairy separator, etc. Inertia manifesting itself in this tendency of the parts of rotating systems to move away from the center of rotation is called *centrifugal force*.

2.5.3 Momentum

The quality of motion possessed by a moving body is defined as the product of the mass and the velocity of the body. It is commonly called *momentum*. Thus a 0.01 kg bullet moving 500 $\frac{\text{m}}{\text{s}}$ has 5 units of momentum. A thousand-kilogram pile driver moving 10 $\frac{\text{m}}{\text{s}}$ has 10,000 units of momentum, etc. We shall always express momentum in M.K.S units, i.e. as a product of kilograms by meters per second.

2.5.4 Second law

Newton's second law is stated thus: *Rate of change of momentum is proportional to the force acting, and takes place in the direction in which the force acts.* While the first law asserted that no change in the momentum of any body takes place unless a force acts upon it, the second law goes a step farther and asserts that two units of force will produce in one second exactly twice as much momentum as does one unit, one half as much as does four unite, etx. Now every one knows from his experience that if he pulls for one second upon a sled, a boat, or any object free to move, the velocity imparted is greater, the greater the pull. That the velocity imparted is *directly proportional* to the pull is the essence of the assertion contained in the second law, and this can be proved only by careful experiments like the following.

let the grooved inclined plane shown in Fig. 22, p. 26 be raised a distance *ab* (Fig. 26), just sufficient to cause the ball to roll down it with uniform velocity. The let the same end be raised 0.2 m higher and the distance which the ball rolls in three seconds be measured with the aid of a metronome, as in 2.4.4. In this case the force which is urging the ball down the incline is the component of the weight of the ball, parallel to the incline. But we proved in 2.2.7 that this is the same fraction of weight of the body that the height of the plane is of its length; e.g. if the length is 5 m the force acting to move the ball is $\frac{20}{500}$, or $\frac{1}{25}$, of the weight of the body. Now let the plane be lifted until d is 40 cm. higher than b. The force is not twice as great as before, since it is $\frac{2}{25}$ of the weight of the ball. Let the stop B (Fig. 22) be placed twice as far down the incline. The ball will be found to reach it again in exactly three seconds.

We learn, then, that doubling the force without changing the mass has doubled the momentum acquired in a given interval of time, since it has doubled the distance which the body has traversed in that length of time. If now we were to double the size of the ball but keep the height of the plane constant, we should find that no change in the velocity acquired per second. This is indeed nothing bu Galileo's experiment which proved that, barring atmospheric resistance, all bodies fall with the same accleration. Hence, since the earth pulls two grams with twice as much force as it pulls one, doubling the mass without changing the velocity involves a coubling of the acting force. The two experiemnts taken together therefore furnish very satisfactory proof of the statement that, whatever be the mass of the body, the momentym acquired by it per second is strictly proportional to the acting force.

ssec:TheNewton

2.5.5 The Newton

Since the gram of force varies somewhat with locality, it has been found convenient for scientific purposes to take the above law as the basis for the definition of a new unit of force. It is called an absolute, or C.G.S unit, because it is based upon the fundamental units of length, mass, and time, and is therefore independent of gravity. It is named the *dyne*, and is defined as *the force which acting for one second upon any body imparts to it one unit of momentum.*

2.5.6 Algebraic statement of the second law

If a force $\vec{F}$ acts for t seconds on a mass of m grams, and in so doing gives it a velocity of $\vec{v}\frac{\mathrm{m}}{\mathrm{s}}$, then the total momentum imparted in a time t is $m\vec{v}$, the momentum imparted per second, we have

$$\vec{F} = \frac{m\vec{v}}{t}. \tag{2.8}$$ f=mv/t

But since $\frac{v}{t}$ is the velocity gained per second, it is equal to the acceleration a. Equation f=mv/t 2.8 may therefore be written

$$\vec{F} = m\vec{a} \tag{2.9}$$ F=ma

This is merely stating in the form of an equation that the force is measured by rate of change in momentum. Thus if an engine, after pulling for thirty seconds on a train having a mass of 2,000,000 kg, has given it a velocity of 0.6 $\frac{\mathrm{m}}{\mathrm{s}}$, the force of the pull is $2,000,000,000\frac{60}{30} = 4,000,000,000$ dynes. To reduce this force to grams we divide by 9.8, and reduce it to kilos we divide further by 1000. hence the pull exerted by the engine on the train $= \frac{400,000,000}{980,000} = 4081$ kg.

2.5.7 Third law

Newton stated his third law thus: *To carry action there is an equal and opposite reaction.* Since force is measured by rate at which momentum changes, this is only another way of saying that whenever one body acquires momentum some other body always acquires an equal and opposite momentum. Thus when a man jumps from a boat to the shore, we all know that the boat experiences a backward thrust; when a bullet is shot from a gun the gun recoils, of "kicks." The essence of the assertion of the third law is that the mass of the man times his velocity is equal to the mass of the boat times its velocity, and that the mass of the bullet times its velocity is equal to the mass of the gun times its velocity. The truth of this assertion has been established by a great variety of careful experiments. The law may be illustrated as follows.

Let a steel ball A (Fig. 27) be allowed to fall from a position C against another exactly similar ball B. In ths impact A will lose all of its velocity and B will move on to a position D which is at the same height at C. hence the velocity acquired by B in the impact is the same as that which A possessed before impact. B has therefore taken away from A exactly the same amount of momentum at A has communicated to B.

It is not always easy to see at first that setting one body into motion involves imparting an equal and opposite motion to some other body. For example, when a gun is held against the earth and the bullet shot upward we are conscions only of the moeion of the bullet. The other body is in theis case the earth and its momentum is the same as that of the bullet. On account, however, of the greatness of the earth's mass its velocity is infinitesimal.

2.5.8 Questions and problems

Add in the questions

Chapter 3

Pressure In Liquids

Pressure_In_Liquids

3.1 Liquid Pressure Beneath a Free Surface

eneath_a_Free_Surface

3.1.1 Proof of the existence of a force beneath the surface of a liquid

e_surface_of_a_liquid

If a long tube closed at the bottom is pushed down into a cylinder of water in the manner shown in Fig. 29, and then left to itself, it will be seen to spring instantly upward.

Evidently, then, the liquid must exert an upward force upon the bottom of the tube. A moment's thought will show that no special experiment was necessary to demonstrate the existence of this force, for a boat or any other body could not float on water if the liquid did not push up against its bottom with sufficient force to neutralize its weight.

3.1.2 Relation between force and depth

tween_force_and_depth

To investigate more fully the nature of this force, we shall use a pressure gague of the form shown in Fig. 30. If the rubber diaphragm whish is stretched across the mouth of the thistle tube A is pressed in lightly with the finger the drop of ink B will be observed to move forward in the tube T, but it will return again to its first position as soon as the finger is removed. If the pressure of the finger is increased, the drop will move forward a greater distance than before. We may therefore take the amount of motion of the frop as a measure of the amount of force acting on the diaphragm.

Now let A be pushed down first 2, then 4, then 8 cm. below the surface. The motion of the index B will show that the force continually increases as the depth increases.

Carefyl quantative measurements made in the laboratory on the exact relation between tyhe force and the depth will show that doubling the depth doubles the force, tripling the depth triples the force, etc.; in other words, that *the force* *is directly proportional to the depth*[1]

To state this relationship algebraically, let F_1 represent the force at some depth D_v and F_2 the force at some other depth D_2; then

$$\frac{F_1}{F_2} = \frac{D_1}{D_2} \tag{3.1}$$

Force_independent_of_direction

3.1.3 Force independent of direction

That there is a lateral as well as a vertical force beneath the surface of a liquid is shown from the fact that water will rush into a boat through a hole in the side as well as through a hole in the bottom.

To compare the amounts of these two forces on a given surface, let the diaphragm A (Fig. 30) be pushed down to some convenient depth, e.g. 10 cm., and the position of the index noted. Then let it be turned sideways so that its plane is vertical (see a, Fig. 30), and adjust in position until its center is exactly 10 cm. beneath the surface, i.e. until the *average* depth of the diaphragm is the same as before. The position of the index will show that the force is also exactly the same as before.

Let the diaphragm then be turned to the position b, so that the gauge measures the *downward* force at a depth of 10 cm. The index will show that this force is again the same.

We conclude, therefore, that *at a given depth a liquid presses up and down and sideways with exactly the same force.*

The_magnitude_of_the_force

3.1.4 The magnitude of the force

In order to determine the exact magnitude of the force exerted by a liquid against a surface, we shall perform a simple experiment with the apparatus shown in Fig. 31.

AB is a thin ground glass plate which is pressed against the bottom of the glass cylinder AD. It is the upward force on the surface AB which we desire to measure. If we pour colored water carefully into the top of the cylinder, the weight of this water will press down on AB and tend to counteract this upward force. When the downward force is equal to the upward force the glass plate AB will drop from the end of the cylinder.

If the plate is thin, so that its own weight is very small, it will be found to frop almost exactly at the instant at which the level oft he water within the culinder is the same as the level of water outside. But at this instant the downward force on AB is evidently the weight of the column of water AFE. Hence the upward force which originally acted on AB was also equal to the weight of the column of water $ABFE$. In other words, *the upward force on any horizontal surface beneath the free surface of a liquid is equal to the weight of a column of water whose base is the given surface and whose height is the depth of the given surface beneath the free surface of the liquid.*

[1] It is recommended that quantative labratory work on the law of depths and on the use of manometers precede this discussion (see e.g. Experiment 5 and 6 of authors' manual).

3.1.5 Magnitude of the force on any surface

In 3.1.3 we proved that the force on a given surface is independent of the direction in which that surface is turned, so long as the dpth of its center is kept the same. Hence, by combining this result with that of 3.1.4, we arrive at the conclusion that the force acting on any surface beneath the free surface of the liquid is equal to the weight of the column of the liquid whose base is the given surface and whose altitude is the *average* depth, i.e. the depth of the center of the surface beneath the free surface of the liquid.

To put this conclusion into algebraic form, let A represent the area of the given surface, h the *mean* depth of the surface beneath the free surface of the liquid, d the density of the liquid, and F the value of the force whish the liquid exerts against the surface A. Then the weight of the column of liquid whose base is A and whose height is h is Ahd (section 1.3.2, page 8). Hence the algebaric statement of the above rule is

$$F = Adh \tag{3.2}$$

3.1.6 The hydrostatic paradox

We may infer from thre preceeding paragraph that the downward force exerted on the bottom of a vessel by a liquid which fills it has nothing whatever to do with the shape of the vessel, but depends only on the area of the base and on the depth and density of the liquid (see formula 3.2). Thus, if the three vessels of Fig. 32 have bases of the same area and are filled to the same depths with liquids of the same density, the forces exerted uon the bases by the liquies should be exactly the same in all three vessels, for by the preceeding paragraph they should all be equal to the weight of a column of liquid of the size $ABCD$.

This comclusion is known as the hydrostatic paradox, because at first sight it seems unreasonable to suppose that the little liquid contained in the third vessel can press down on the bottom with the same force as the large amount of liquid contained in the second vessel. The following experiment, however, will furnish a complete demonstration of the correctness of the conclusion, and will prove expreimentally that the downward force on the bottom of the vessel has nothing to do with the shape of the vessel.

Let the funnel ABD (Fig. 33, (1)) be closed at the bottom by the some glass plate whish was used in the experiment of Fig. 31. At a given depth beneath the free surface of the liquid the upward force acting against the lower side of the plate AB must, of course, be the same as it was before, then the cylinder was used, i.e. it is equal to the weight of the column of water $ABEF$ (3.1.4). Now let water be poured carefully into the top of the funnel until the plate AB is forced off. Just as in the experiment of section 3.1.4, this will be found to occur exactly when the level of the water inside of the funnel has risen to the height of the water outside. Hence the liquid within the funnel ABD must exert the same downward force on AB as sis the liquid within the cylindrical tube $ABEF$ in the experiment of section 3.1.4.

Pressure

3.1.7 Explanation of the hydrostatic paradox

A moment's consideration will show that there is no real inconsistency in the fact that the third vessel of Fig. 32 exerts a force on the bottom so much greater than its own weight, and that the second vessel exerts a force so mush less than its own weight. For the law sdiscovered in section Force_independent_of_direction 3.1.3, that the force at a given depth beneath the free surface of a liquid acts equally in all directions upon all equal surfaces, means that while the liquid in the third vessel does indeed exert a downward force on AB which is equal to the weight of the column of water $ABCD$, it also exerts an upwards force on the surfaces of af and eb which is equal to the weight of the water which would fill the spaces of $afhC$ and $ebDg$. Hence the net or resultant force which is acting down is the difference between the downward force on AB and the upward forces of af and eb, and this will be seen at once from the figure to be simply the weight of the liquid in the vessel, as of course it must be.

Similarly in the second vessel of Fig. 32, while the force acting directly upon te bottom is only the weight of the column of water $ABCD$, the downward force upon the sides Am and Bn amounts, in all, exactly to the weight of the remainder of the water in the vessel, i.e. to the weight of the water contained in the spaces AmC and BnD.

3.1.8 Pressure in liquids

Thus far attention has been confined to the total force exerted by a liquid against the *whole* of a given surface. It is often more convenient to consider the surface divided into square centemeters and to confine the attention to the force exerted upon one of these square centermeters. In physics the word "pressure" is used exclusivly to denote this *force per unit area.* Thus, if the weight of the column of liquid $ABCD$ in Fig. 32 is 0.1 g, and if the ara of the surface AB is 0.2 m^3., then the force per square centemeter acting on AB is 5 g Hence we say that the *pressure* on AB is 5 g. Pressure is thus seen to be a measure of the *intensity* of the force acting on a surface, and not to depend at all upon the *area* of the surface.

It is clear, then, that in order to obtain pressure, we divide the total force acting by the area of the surface against shich it acts. Or, algebraically stated, if we represent pressure by p, force by F, and area by A, we have

$$p = \frac{F}{A} \tag{3.3}$$ p=F/A

In other words, *the liquid pressure existing at any depth h beneath the free surface of any liquid of density d is equal to the product of this depth by the density of the liquid*; i.e. it is the weight of the column of liquid whose height is equal to the given depth, and the area of whose cross section is unity. it is important to remember this technical use of the word "pressure."

3.1.9 Levels of liquids in connection vessels

in_connection_vessels

It is a perfectly familiar fact that when water is poured into a teapot it stands at exactly the same level in the spout as in the body of the teapot; or if it is poured into a number of connected tubes, like those shoen in Fig. 34, the surfaces of the liquid in various tubes lie in the same horizontal plane. These facts follow as a necessary consequence of the law, discovered above, the pressure beneath the surface of a liquid depende simply upon the *depth* and not at all upon the shape and size of the vessel.

Thus, in accordance with the above rule, in Fig. 35 the pressure acting at o to drive water to the left is equal to the density of the liquid times the height hs; and the pressure acting at e to drive water to the right is equal to the same density times the height eg. hence these two pressures will be balanced and the liquids will be at reat only when these two heights are the same, i.e. then the free surfaces in the two vessels are in the same horizontal plane.

If water is poured in at s so that the height hs is increased, the pressure to the left at o becomes greater than the pressure to the right at e, and a flow of water takes plase to the left until the heights are again equal.

3.1.10 Questions and problems

Add in these questions

3.2 Pascal's Law

3.2.1 Transmission of pressure by liquids

From the fact that pressure within a free liquid depends simply upon the depth and density of the liquid, it is possible to deduce a very suprising conclusion, which was first stated by the famous French scientist, mathematician, and philsopher, Pascal (1623-1662).

Let us imagian a vessel of the shape shown in Fig. 36, (1), to be filled with water up to the level ab. For simplicity let the upper portion be assumed to be 1 sq. cm. in cross section. Since the density of water is 1, the force with shich it presses against any square centemeter of the interior surface which is h cm. beneath the level ab is h grams. Now let one gram of water (i.e. 1 cc.) be poured into the tube. Since each square centimeter of surface which was before h cm. beneath the level of water in the tube is no $h+1$ cm. beneath this level, the new pressure which the water exerts against it is $h+1$ g.; i.e. applying 1 g. of force to the square centimeter of surface ab has added 1 g. to the force exerted by the liquid against each square centimeter of the interior of the vessel. Obviously it can make no difference whether the pressure which was applied to the surface ab was due to the weight of the water or to a piston carrying a load, as in Fig. 36, (2), or to any other cause whatever. We thus arrive at Pascal's conclusion that *pressure applied anywhere to a body of confined liquid is transmitted by the* *liquid so as the act with undiminished force on every square centimeter of the containing vessel.*

3.2.2 Multiplication of force by the transmission of pressuer by liquids

Pascal himself pointed out that with the aid of the principle stated above we ought to be able to transform a very small force into one of unlimited magnitude. Thus if the area of the cylinder ab, Fig. 37, is 1 cm^2, while that of the cylinder AB is 1000 cm^2, a force of 1 kg. applied to ab would be transmitted by the liquid so as to act with a force of 1 kg. on each square centimeter of the surface AB. Hence the total upward force exerted against the piston AB by the one kilo applied at ab would be 1000 kg. Pascal's own words are as follows: "A vessel full of water is a new principle in mechanics, and a new machine for the multiplication of force to any required extent, since one man will by this means be able to move any given weight."

3.2.3 The hydraulic press

The experimental proof of the correctness of the conclusions of the preceeding paragraph is furnished by the hydraulic press, an instrument now in common use for subjecting to enormous pressures paper, cotton, etc.; for punching holes through iron plates, testing the strength of iron beams, extracting oil from seeds, making dies, embossing metal, etc.

Such a press is represented in section in Fig. 38. As the small piston p is raised, water from the cistern C enters the piston chamber through the valve v. As soon as the down stroke begins the valve v closes, the valve v' opens, and the pressure applied on the piston p is transmitted through the tube K to the large reservior, where it acts on the large cylinder P with a force which is as many times that applied to p as the area of P is times the area of p.

hand presses similar to that shown in Fig. 39 are often made which are capacle of exerting a compressing force of from 500 to 1000 tons.

3.2.4 No gain in the product of force times distance

It should be noticed that, while the force acting on AB (Fig. 37) is 1000 times at great as the force acting on ab the distance through which the piston AB is pushed up in a given time is but $\frac{1}{1000}$ of the distance which piston ab moves down. For, forcing ab down a distance of 1 cm. crownds but 1 cc. of water over into the large cylinder, and this accitional cubic centimeter can raise the level of water there but $\frac{1}{1000}$ cm. We see therefore that the product of the force acting by the distance moved is preciesly the same at both ends of the machine. This important conclusion will be found in out future study to apply to al l machines.

3.2.5 The hydraulic elevator

Another very common application of the principle of transformation of pressure by liquids is found in the hydraulic elevator. The simplist form of such an elevator is shown in Fig. 40. The cage A is borne on the top of a long piston P which runs in a cylindrical pit C of the same depth as the height to which the carriage must ascend. Water enters the pit either directly from the water mains m of the city's supply, or, if this does not furnish sufficient pressure, from a special reservoir on top of the building. When the elevator boy pulls *up* on the cord cc, the valve v opens so as to make connection from m into C. the elevator then ascends. When cc is pulled *down*, v turns so as to permit the water in C to escape into the sewer. The elevator then descends.

Where speed is required the motion of the cylinder is communicated indirectly to the cage by a system of pullys like that shown in Fig. 41. With this arrangement a goot of upward motion of the cylinder P causes the counterpoise D of the cage to descend 2 m, for it is clear from the figure that when the cylinder goes up 1 m enough rope must be pulled over the fixed pulley p to lengthen each of the two strands a and b 1 m Similarly, when the counterpoise descends 2 m the cage ascends 4 m Hence the cage moves four times as fast and four times as far as the cylinder. The elevators in the Eiffel Tower in Paris are of this sort. They have a total travel of 420 m and are capable of lifting 50 people 400 m per minute.

3.2.6 City water supply

Fig. 42 illustrates the method by which a city is often supplied with water from a distant source. The aqueduct from the lake a passes under a road r, a brook$b$ a hill H, and into a reservoir e, from which it is forced by the pump p into the standpipe P, whence it is distributed to the houses of the city. if a static condition prevailed in the whole system, then the water level in e would of necessity be the same as that of a, and the level in the pipes of the building B would be the same as that in the standpipe P. But when the water is flowing the friciton of the mains causes the level e to be souewhat less than that in a, and that in B less than that in P. It is on account of the friction both of the air and of the pipes that the fountain f does not actually rise nearly as high as the ideal limit shown in the figure (see dotted line).

3.2.7 Artesian wells

It is in the principle of transmission of pressure by liquids that artesian wells find their ecplanation. Fig. 43 is an ideal section of what geologists call an artesian basin. The stratum A is composed of some porous material such as cand, open-textured sandstone, of broken rock, through which the water can precolate easily. Above and below it are strata C and B of clay, slate, or some other material impervious to water. The porous layer is filled with water which finds enterance at the outcropping margins. As soon as boring is made through the layer C the water gushes forth because of the transmission of pressure from higher levels. A well of this sort exists near kissingen, Germany, which is 1800 ft. deep and which throws a stream of water 58 ft. high. The deepest artesian wells have been bored in the desert of Sahara and an abundant water supply found at a depth of 200 ft. Great numbers of artesian wells exist in the United States. Notable ones are located at Chicago, Louisville (Kentuckey, and Charleston (South Carolina). The artesian basins in which the wells are found are often a hundred miles or more in width.

3.2.8 Questions and problems

Add in these questions

3.3 The Principle of Archimedes

The_Principle_of_Archimedes

[2]

3.3.1 Loss of weight of a body in a liquid

The preceding expreiments have shown that an upward force acts against the bottom of any body emmersed in a liquid. If the body is a boat, cork, piece of wood, or any body which floats, it is clear that, since it is in equilibrium, this upward force must be equal to the weight of the body. Even if the body does not float, everyday observation shows that it still loses a portion of its natural weight, for it is well known that it it easier to lift s stone uncerwater than in air; or again, that a man in a bath tub can support his whole weight by pressing lightly against the bottom with his fingers. It was indeed this vary ovservation whish first led the old Greek philsopher, Archimedes (287-212 B.C.), to the discovery of the exact law which governs the loss of weight of a body in liquid. Hiero, the tyrant of Syracuse, had ordered a gold crown made, by suspected that the artisan had fraduently used silver as well as gold in its construction. He ordered Archimedes to discover whether or not this were true. How to do so without destroying ths crown was at first a puzzle to the old philsopher. While in his daily bath, noticed the loss of weight of his own body, it suddenly occurred to him that *any body immersed in a liquid must lose a weight equal to the weight of the didplaced liquid*. He is said to have jumped at once to his feet and rushed through the streets of Syracuse crying, “Eureka, eureka!” (I have found it, I have found it!)

3.3.2 Theoretical proof of Archimedes’ principle

l_proof_of_Archimedes’_principle

It is probable that Archimedes, with that faculty which is so common among men of great genius, saw the truth of this conclusion without going through

[2] A laboratory exercise on the experimental proof of Archimedes’ principle should precede this discussion. See e.g. Experiment 7 of the authors’ manual.

any logical process of proof. Such a proof, however, can easily be given. Thus, since the upward force on the bottom of the block *abcd* (Fig. 45) is equal to the weight of the column of liquid *obce*, and since the downward force on the top of this block is equal to the weight of the column of the liquid *oade*, it is clear that the upward force must exceed the downward force by the weight of the column of liquid *abcd*; i.e. *the bouyant force exerted by the liquid is exactly equal to the weight of the displaced liquid*.

The reasoning is exactly the same no matter what may be the nature of the liquid in which the body is immersed, not how far the body may be beneath the surface. Further, if the body weighs more than the liquid which is displaces, it must sink, for it is urged down with the force of its own weight, and up with the lesser force of the weight of the displaced liquid. But if it weighs less than the displaced liquid, the the upward force due to the displaced is greater than its own weight, and consequently it must rise to the surface. When it reaches the surface the downward force on the top of the block, due to the liquid, becomes zero. The body must, however, continue to rise until the upward force on its bottom is equal to its own weight. But this upward force is always equal to the weight of the displaced liquid, i.e. to the weight of the column of liquid *mben* (Fig. 46).

Hence *a floating body must displace its own weight of the liquid in which it floats*. This statement is embraced in the original statement of Archimedes' principle, for a body which floats has lost its whole weight.

3.3.3 Experimental proof of archimedes' principle

To test experimentally the truth of Archimedes' principle, we weigh a body of known volume first in air, then in some liquid (Fig. 47). If the principle is correct, the difference should be exactly equal to the product of the volume of the body by the density of the liquid, since this product is the weight oof the displaced liquid. If the liquid is water of density 1, then the loss of the weight should be numerically equal to the volume of the body.

To test the principle for a floating cylinder like that shown in (Fig. 49. If the liquid is water, this volume whould be numerically equal to the weight of the floating cylinder. Tests of this sort are best performed by the pupil in the laboratory.

3.3.4 Density of a heavy solid

The density of a body is by definition its mass divided by its volume. It is always possible to obtain the mass of a body by weighing it, but it is not, in general, possible to obtain the volume of an irregular body from measurements of its dimensions. Archimedes' principle, however, furnishes an accurate and easy method for obtaining the volume of any solid, however irregular, for by the preceeding paragraph this volume is *numerically* equal to the loss of weight in water. Hence the equation which defines density, namely,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

becomes in this case

$$\text{Density} = \frac{\text{Mass}}{\text{Loss of weight in water}}. \tag{3.4}$$

3.3.5 Density of a solid lighter than water

If the body is too light to sink of itself, we may still obtain its volume by forcing it beneath the surface with a sinker. Thus suppose w_1 represents the weight on the right pan of the balance when the body is in air and the sinker in water, as in Fig 48; while w_2 is the weight on the right pan when both the body and sinker are under water. Then $w_1 - w_2$ is obviously the bouyant effect of the water on the boyd alone, and is therefore equal to the weight of the displaced water which is numerically equal to the volume of the body.

3.3.6 Density of liquids by hydrometer method

Archimedes' principle also furnishes an easy method for finding the density of any liquid. For suppose a uniform cylinder like that of Fig. 49 is floated in water and is found to sink a distance l_1; then, if$A$ represents the area of the cross section of the cylinder, the volume of the displcemed water is 1, the weight of the displaced water is also Al_1. By Archimedes' principle this is equal to te weight of the floating body. Next suppose that the same cylinder is floated in the liquid whose density d_2 is sought [Fig. 49, (2)]. It will now sink some distance l_2. The volume of the displaced liquid will be Al_2, and its weight will be Al_2d_2. By Archimedes' principle this is again equal to the weight w of the floating body. Hence

$$Al_2d_2 = Al_1, \text{ or } d_2 = \frac{l_1}{l_2}; \tag{3.5}$$ □

i.e. *the density of the unknown liquid is simply the ratio of the depth l_s, which the cylinder sinks in water, to the depth $._s$, which it sinks in the unknown liquid.*

3.3.7 commercial form of hydrometer

The commercial constant-weight hydrometer such as is now in common use for testing the density of alchol, milk, acids, sugar colutions, etc., instead of being a cylinder like sholn in Fig. 49, is of the form shown in Fig. 50. The stem is calibrated so that the density of any liquid may be read upon it directly. The advantage of this form over that of Fig. 49 is that it is much more suitable for the detecting of very slight differences between the densities of two liquids. The reason for this will be clear when it is remembered that the instrument must always sink until is displaces its own weight of the liquid, and that if the stem is made very narrow in comparison withthe lower portion, the sinking of the considerable portion of the stem will add but very little to the total volume of the liwuid displaced. By making the cylinder exceedingly long the same sensitivity could of course be obtained with the cylindrical form, but it would then by inconvenient to use.

3.3.8 Density of liquids by "loss-of-weight" method

If any heavy body is weighed first in air, then in water, and lastly in a liquid of unkowwn density d_2, then, since the weight of the water displaced by the body is its volume V times its density 1, and since the weight of the unknown liquid displaced is the same as V times the density d_2, we have by Archimedes' principle, if L_1 represents the loss of weight in water and L_2 the loss in the unknown liquid,

$$L_1 = V \cdot 1, \text{ and } L_2 = V d_2.$$

Dividing the second equation by the first gives

$$d_2 = \frac{L_2}{L_1}; \tag{3.6}$$

i.e. the density of the unknown liquid is the loss of weight in that liquid by the loss of weight in water.[3]

3.3.9 Questions and problems

Add in these questions

[3]Labratory experiments in determination of densities of solids and liquids shoulc follow or accompany the discussion of this chapter. See e.g. Experiments 8 and 9 of the authors' manual.

Chapter 4

Pressure In Air

4.1 Barometric Phenomena

4.1.1 The weight of air

To ordinary observation air is scarcely perceptible. It appears to have no weight and to offer no resistance to bodies passing through it. But if a bulb be balanced as in Fig. 54, then removed and filled with air under pressure by a few strokes of a bycle pump, it will be found when again placed on the balance, to be heavier than it was before. On the other hand, if the bulb be connected with an air pump and exhausted, it will be found to have lost weight. Evidently, then, air can be put into and taken out of a vessel, weighed, and handled, just like a liquid or solid.

We are accustomed to say that bodies are "as light as air," yet careful measurement shows that it takes but 12 ft^3 of air to weigh a pound, so that a single largh room contains more air than an ordinary man can lift. Thus the air in a room 60 ft. by 30 ft. by 15 ft. weighs more than a ton. The exact weight of air at the freezing temperature and undernormal atmospheric conditions is 0.001293 g per c^3, i.e. 1.293 g per liter.

4.1.2 Proof that air exerts pressure

Since air has weight, it is to be infered that it, like a liquid, exerts forces against any surface immersed in it. The following experiments prove this.

Let a rubber membrane be stretched over a glass vessel, as in Fig. 55. As the air is exhausted from beneath the membrane the latter will be observed to be more and more depressed until it will finally burst under the pressure of the air above.

Again, let a tin can be partly filled with water and the water boiled. The air will be exprelled by the escaping steam. While the boiling is still going on let the can be tightly corked, then placed in a sink or tray and cold water poured over it. The steam will be condensed and the weight of the air outside will crush the can.

se_of_liquids_in_exhausted_tubes

4.1.3 Cause of the rise of liquids in exhausted tubes

If the lower end of a long tube be dipped into water and the air exhausted from the upper end, water will rise in the tube. We prove the truth of this statement every time we draw lemonade through a straw. The old Greeks and Romans explained such phenomena by saying that "nature abhors a vaccum," and this explanation was still in vogue in Galileo's time. But in 1640 the Duke of Tuscany had a deep well dug near Florence, and found to his suprise that no water pump which could be obtained would raise the water higher than 32 feet above the level of the well. When he applied to the aged Galileo for an explanation the latter replied that evidently "nature's horror of a vaccum did not extend beyond thirty-two feet." It is quite likely that Galileo suspected that the pressure of the air was responsible for the phenomenon, for he had himself proved before that air has weight, and, furthermore, he at once devised another experiment to test, as he said, the "power of a vaccum." He died in 1642 before the ecperiment was performed, but suggested to his pupil, Torricelli, that he coutinus the investigation.

4.1.4 Torricelli's experiment

Torrricelli argued that if water would rise 32 ft., then mercury, which is about 13 times as heavy as water, ought to rise but $\frac{1}{13}$ as high. To test this inference he performed in 1643 the following experiment.

Let a tube about 4 ft. long, which is sealed at one end, be completly filled with mercury, as in Fig. 56, (1), then closed with the thumb and inverted, and the bottom of the tube then immersed in a dish of mercury, as in Fig. 56, (2). When the thumb is removed from the bottom of the tube, the mercury will fall away from the upper end of the tube in spite of the fact that in so doing it will leave a vaccum abot it, and its upper surface will in fact stand about $\frac{1}{13}$ of 32 ft., i.e. 29 or 30 in, above the mercury in the dish.

Torricelli concluded from his experiment that the rise of liquids in exhausted tubes is due to an outside pressure exerted by the atmosphere on the surface of the liquid, and not any mysterious sucking power created by the vaccum.

4.1.5 Further decisive tests

An unanswerable argument in favor of this conclusion will be furnished if the mercury in the tube falls as soon as the air is removed from above the surface of the mercury in the dish.

To test this point, let the dish and tube be placed on the table of an air pump, as in Fig. 57, the tube passing through a tightly fitting rubber stopper *A*, in the bell jar. As soon as the pump is started the mercury in the tube will, in fact, be seen to fall. As the pumping is continued it will fall nearer and nearer to the level in the dish, although it will not usually reach it for the reason that an ordinary vaccum is not capable of producing as good a vaccum as that which exists in the top of the tube. As the air is allowed to return to the bell jar the mercury will rise in the tube to its former level.

4.1.6 Amount of atmospheric pressure

Torricelli's ecperiment shows exactly how great the atmospheric pressure is, since this pressure is able to balance a column of mercury of definite length. In accordance with Pascal's law the downward pressure exerted by the atmosphere on the surface of the mrecury in the dish (Fig. 58) is transmitted as an exactly equal upward pressure on the layer of mercury inside the tube at the same level as the mercury outside. But he downward pressure at this point iwthint he tube is equal to hd, where d is the density of mercury and h is the depth below the surface b. Since the average height of this column at sea level is found to be 76 cm., and since the density of mercury is 13.6, the downward pressure inside the tube at a is equal to 76 times 13.6 or 1033.6 g. per sq. cm. Hence the atmospheric pressure acting on the surface of mercury in the dish is 1033.6 g., or roughly 1 kg., per sq. cm. This amounts to about 15 lb. per sq. in.

4.1.7 Pascal's experiment

Pascal thought of another way of testing whether or not it were indeed the weight of the outside air which sustains the column of mercury in an exhausted tube. He reasoned that, since the pressure in a liquid diminiches on ascending toward the surface, atmospheric pressure ought also to dimish on passing from sea level to mountain top. As no mountain existed near Paris, he carried Toricelli's apparatus to the top of a high tower and found, indeed, a slight fall in the height of the column of mercury. He then wrote to his brother-in-law perrier, who lived near Puy de Dome, a mountain in the south of France, and asked him to try the experiment on a larger scale. Perrier wrote back that he was "ravished with admiration and astonishment" when he found that on ascending 1000 m. the mercury sank about 8 cm. in the tube. This was in 1648, five years after Torricelli's discovery.

At the present day geological parties actually ascertain differences in altitude by observing the change in barometric pressure as they ascend of descend. A fall of 1 mm. in the column of mercury corresponds to an ascent of about 12 m.

4.1.8 The barometer

The modern barometer (Fig. 59) is essentially nothing more nor less than Torricelli's tube. Taking a barometer reading consists simply in accurately measuring the height of the mercury column. This height varies from 73 to 76.5 cm. in localities which are not far above sea level, the reason being that disturbances in the atmosphere affect the pressure at the earth's surface in the same way in which eddies and high waves in a tank of water would affect the liquid pressure at the bottom of the tank.

The barometer does not directly foretell the weather, but it has been found that a low or rapidly falling pressure is usually accompanied, of soon followed, by stormy conditions. Hence the barometer, although not an infallible weather prophet, is nevertheless of considerable assistance in forcasting weather condition some hours ahead. Further, by comparing at a central station the telographic reports of marometric reading made every few hours at stations all over the country, it is possible to dtermine in what direction the atmospheric eddies which cause barometer changes and storym conditions are traveling, and hence to "forecast" the weather even a day or two in advance.

4.1.9 The first barometers

Torricelli actually constructed a barometer not essentially different from that shown in Fig. 59, and used it for observing changes in the atmospheric pressure; but perhaps the most interesting of the early barometers was that set up about 1650 byt he famous old German physicist Otto von Guericke of Magdeburg (1602-1686). He used for his barometer a water column the top of which passed through the roof of his house. A wooden image image which floated on the upper surface of the water appeared abouve the house top in fair weather but retired from sight in foul, a circumstance which led his neighbors to charge him with being in league with Satan.

4.1.10 Effect of inclining a barometer

If a barometer tube is inclined i the manner shown in Fig. 60, the top of the mercury will be found to remain always in the same horizontal plane. Explain, remembering that pressure equals height times density (Fig. 35).

4.1.11 The anerooid barometer

Since the mercurial barometer is somewhat long, and inconvenient to carry, geological and surveying parties commony use an instrument called the *aneroid barometer* (Fig. 61). It consists of essentially of an air-tight cylinderical box D, the tops of which is a metallic diaphragm which bends slightly under the influence of change in the atmospheric pressure. This motion of the top of the box is multiplied by the delicate system of levers and communicated to the hand B which moves over a dial whose reading are made to correspond to the readins of a mercury barometer. These instruments are made so sensitive as to indicate a change in pressure when they are moved no farther than from a table to the floor.

4.1.12 Questions and problems

Add in these questions

4.2 Compressibility and Expansibility of Air

4.2.1 Incompressibility of liquids

Thus far we have found very striking resembalances between the conditions which exist at the bottom of a body of liquid and those which exist at the bottom of the great ocean of air in which we live. We now come to a most important difference. It is wel lknown that if two liters of water be poured into a tall cylindrical vessel, the water will stand exactly twice as high as if the vessel contain but one liter; of it ten liters be poured in, the water will stand ten times as high as if there be but one liter. This obviously means that the lowest liter in the vessel is not measurably diminished in volume by the weight of as many as nine liters of water resting upon it.

It has been found by farefully devised ecperiments that compressing weights enormously greater than thesse may be used without producing a marked effect; e.g. when a cubic centimeter of water is subjected to the stupendous pressure of 2,000 kilogram., its volume is reduced to due 0.90 cm^3 Hence we say that water, and liquids generally, are practically incompressible. Had it not been for this fact we should not have been justified in taking the pressure at any depth below the surface of the sea as tie simple product of the depth by the density at the surface.

4.2.2 Compressibility of air

When we study the effects of pressure on the air we find a wholly different behavior from that described above for water. It is very eary to compress a body of air to one half, one fifth, or one tenth of its normal volume, as we prove every time we inflate a pneumatic tire or cushion of any sort. Further, the *expansibility* of air, i.e. its tendency to spring back to a larger volume as soon as the pressure is relieved, is proved every time a tennis ball or football bounds, or the cork is driven from a popgun.

But it is not only air which has been crowded into a pneumatic cushin by some sort of pressure pump which is in this state of readiness to expand as soon as the pressure is diminished. The ordinary air of the room will expand in the same way if the pressure to which it is subjected is relieved.

Thus let a bladder of toy balloon be filled with air under ordinary conditions and then tied up air-tight and placed under the reciver of an air pump. As soon as the pump is set into operation the inside air will expand with sufficient force to butst the bladder, or to greatly distend the balloon, as shown in Fig. 66.

Again, let two bottles be arranged as in Fig. 67, one being stoppered air-tight, while the other is uncorked. As soon as the two are placed under the receier of an air pump and the air exhausted, the water in A will pass over into B. When the air is readmitted to the receiver the water will flow back. Explain.

4.2.3 Why hollow bodies are not crushed by atmospheric pressure

The preceeding experiments show why the walls of hollow bodies are not crushed in by the enormous forces which the weight of the atmosphere exerts against them. For the air inside such bodies presses their walls out with as much force at the outside presses them in. In the experiment of section Cause_of_the_rise_of_liquids_in_exhausted_ 4.1.3 the inside air was removed by the excaping steam. When this steam was condensed by the cold water, the inside pressure became very small and the outside pressure then crushed the can. In the experiment shown in Fig. 66 it was the outside pressure which was removed by the air pump, and the pressure of the inside air then burst the bladder.

4.2.4 Boyle's law

The first man to investigate the exact relation between the charge in the pressure exerted by a confined body of air and its change in volume was Robert Boyle, an Irishman (1626-1691). We shall repeat a modified form of his experiment much more carefully in the laboratory; but the following will illustrate the method by which he discovered one of the mose important laws of physics.

Let mercury be poured into a bent glass tube until it stands at the same level in the closed arm AC as in the open arm BD (Fig. 68). There is now confined in AC a certain volume of air under the pressure of one atmosphere. Call this pressure P_1. Let the length of AC be measured and called V_1. Then let the mercury be poured into the long arm until the level in this arm is as many centimeters above the level in the short arm as there are centimeters in the barometer height. The confined air is now underpressure ot two atmospheres. Call it P_2. let the new volume $A_1C(=V_2)$ be measured. It will be found to be just half its former value.

Hence we learn that doubling the pressure exerted uopon a body or air halves its volume. If we tripled the pressure, we should have found the volume reduced to one third its initial value, etc. Hence, *the pressure which is given quantity of air at constant temperature exerts against the walls of the containing vessel in inversely proportional to the volume occupied*. This algebraically stated thus

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}, \text{ or } P_1V_1 = P_2V_2. \tag{4.1}$$

P1V1=P2V2

This is Boyle's law. It may also be stated in slightly different form. Doubling, tripling, or quadrupling the pressure must double, triple, or quadruple the *density*, since the volume is made only one half, one third, or one fourth as much, whild the mass remains unchanged. Hence *the pressure which air exerts is directly proportional to its density, or, algebraically,*[1]

$$\frac{P_1}{P_2} = \frac{D_1}{D_2} \tag{4.2}$$

P1/P2=D1/D2

[1] A laboratory experiment on Boyle's law should follow this discussion. See e.g. Experiment 10, authors' manual.

4.2.5 Extent and character of the earth's atmosphere

From the facts of compressibility and expansibility of air we may know that the air, unlike the sae, must become less and less dense as we ascend from the bottom toward the top. Thus at the top of Mount Blanc, where the barometer height is but 38 cm., or on half of its value at sea level, the denisty also must, by Boyle's law, be just one half as much as at sea level.

No one has ever ascended higher than 7 mi., which was approximately the hight attained in 1862 by the two daring English aëronauts, Glasier and Coswell. At this altitude the barometric height is but about 7 in. and the temperature about −60 degrees F. Both aëronauts lost the use of their limbs and Mr. Glasier became unconscious. Me. Coxwell barely succeeded in grasping with his teeth the rope which opened a valve and caused the balloon to descend. Again, on July 31, 1901, the French aëronaut M. Berson rose without injury to a height of about 7 mi. (35,420 ft.), his success being due to the artificial inhalation of oxygen.

By sending up self-registereing thermometers and barometers in the ballons which burst at great altitudes, the instruments being protected by parachutes from the dangers of rapid fall, the atmosphere has been explored to a height of 22,290 m. (13.8 mi.), this being the height attained on December 4, 1902, by a little rubber balloon 76 im. in diameter which was sent up from the strasburg (Germany) observatory. These extreme heights are calculated from the indications of the self-registering barometers. Fig. 69 shows, in the right-hand column, the densities of air at various heights in terms of its density at sea level. In the next column are shown the corresponding barometer heights in inches, while the left-hand column indicates heights in miles.

It will be seen that at a heights of 35 mi. the density is estimated to be but $\frac{1}{30000}$ of its value at sea level. By calculatinng how far below the horizon the sun must be when the last traces of color disappear from the sky, we find that at a height as great as 45 mi. there must be air enough to reflect some light. How far beyond this an extremely rarefied atmosphere may extend, no one knows. It has been extimated at all the way from 100 to 500 mi. These estimates are based on observations of the height at which meteors first became visible, on the height of the aurora borealis, and on the darkening of the surface of the moon just before it eclipsed by the shadow of the solid earth.

4.2.6 Height of the "homogeneous atmosphere"

Although, then, we cannot tell to what height the atmosphere extends, we do know with certainty that the weight of a column of air is 1 sq. cm. in crossection and reaching from earth's surface to the extreme limits of the atmosphere will just balance a column of mercury 76 cm. high, for this was shown by Torricelli's experiment. Since 1 cm^3 of air at the earth's surface weighs about 1.2 mg., i.e. since the density of air is about 0.00212, of on eight-hundreth that of water, and since mercury is about 13.6 times as heavy as water, it follows that if the air had the same density at all latitudes which it has at the earth's surface, its height would be $76 \cdot 13.6 \cdot 800 \approx 8.27$ km. The tops of the Himalayas would therefore rise above it. This height of 5 mi., which is the height to which the air would extend if it, like the ocean, had the same density throughout, is called the *height of the homogeneous atmosphere*.

4.2.7 Density of air below sea level

The same cause which makes air diminish rapidly in density as we ascend above sea level must produce a rapid increase in its density as we descend below sea level. It has been calculated that if boring could be made in the earth 35 mi. deep, the air at the bottom would be one thousand times as dense as at the earth's surface. Therefore wood and even water would float in it.

4.2.8 Questions and problems

Add this section

4.3 Pneumatic Appliances

4.3.1 The siphon

Let a rubber of glass tube be filled with water and then placed inthe position shown in Fig. 72. Water will be found to flow through the tube from vessel A into vesel B. If, then. B be raised until the water in it is at a higher level than that in A, the direction of flow will be reversed. This instrument, shich is called a siphon, is very useful in removing liquids from vessels which cannor be overturned, or for drawing off the upper layers of liquid withour disturbing the lower layers.

Ths explanation of the siphon's action is readily seen in Fig. 72. Since the tube acb is full of water, water, must evidently flow through it if the force which pushes it one way is greater than that which pushes it the other way. Now the upward pressure at a is equal to atmospheric pressure minus the downward pressure due to the water column ad; whild the upward pressure at b is the stmospheric pressure minus the downward pressure due to the water column be. Hence the pressure at a exceeds the pressure at b by the pressure due to the water column $fb = 0$, and the forces acting at the two ends of the tube are therefore equal and opposite. it will also cease to act when the bend c is more than 34 ft. above the surface of the water in A, since then a vacuum will form at the top. atmospheric pressure being unable to raise water to a height greater than this in either tube.

Would a siphon flow in a vaccum?

4.3.2 The intermittent siphon

Fig. 73 represents an intermittent siphon. If the vessel is at first empth, to whatlevel must it be filled before the water will flow out at o? To what level will the water then fall before the flow will cease?

The intermittent spring sometimes found in nature is nothing but a natural siphon of this kind. Its action may be understood from Fig. 74.

4.3.3 The aspirating siphon

It is clear from the theory of siphon action that the flow cannot start unless the tube is initially full of the liquid. Fig. 75 represents a so-called aspirating siphon, an instrument designed to minimize the inconvenient and danger incident upon starting the flow when it is desired to siphon off acids or other disagreeable or poisonous liquids. The open b is first closed; the tube is the filled by sucking on the end O while the end c is immersed in the liquid to be siphoned off. The bulb E is made so large that there is no danger of inadvertently sucking liquid into the mouth.

4.3.4 The air pump

The air pump was invented in 1650 by Otto von Guericke, mayor of Magdeburg, Germany[2], who deserves the greater credit, since he was apparently wholly without knowledge of the discoveries which Galileo, Torricelli, and Pascal had made a few years earlier regarding the character of the earth's atmosphere. A simple form of such a pump is shown in Fig. 76. When the piston is raised the air from the receiver R expands into the cylinder B through the valve A. When the piston descents it compresses this air, and thus closes the valve A and opens the exhaust valve C. Thus with each double stroke a certain fraction of the air in the receiver is transformed from R through the cylinder to the outside.

In many pumps the valve C is in the piston itself.

4.3.5 The compressor pump

A compression pump is nothing by an exhaust pump with the valves reversed, so that A closes and C opens on the upstroke, and A opens and C closes on the downstroke. In its cheaper forms, e.g. the common bicycle pump, the valve C is often replaced by a very simple device called a cup valve. This valve consists of a disk of leather a little larger than the barrel of the pump, attached to a loosely fitting metal piston. When the piston is raised the air passes in around the leather, but when it is lowered the leather is crowded closely against the walls, to that there is no escape for the air (Fig. 77).

Compressed air finds many applications in such machines as air drills (used in mining), air brakes, air motors, etc., that the compression pummp must be looked upon as of much greater importance industrially than the exhaust pump.

[2] Cite source.

4.3.6 The lift pump

The common water pump shown in Fig. 78, has been in use at least since the time of Aristotle (fourth century B.C.). It will be seen from the figure that it is nothing more nor less than a simplified form of air pump. In fact, in the earlier strokes we are simply exhausting air from the pipe below the valve b. Water soulc never be obtained at S, even with a perfect pump, if the valve b were not within 34 ft. of the surface of the water in W. Why? On account of mechanical imperfections this limit is usually about 28 ft. instead of 34. Let the student analyze, stroke by stroke, the operation of pumping water from a well with the pump of Fig. 78. Why will pouring in a little water at the top, i.e. "priming," often assist greatly in starting such a pump?

4.3.7 The force pump

Fig. 79 illustrates the construction of the force pump, a device commonly used when it is desired to deliver water at a point higher than the piston at which it is convenient to place the pump itself. Let the student analyze the action of the pump from a study of the diagram.

It will be seen that the discharge from such an arrangement as that shown in Fig. 79 must be intermittent, since no water can flow up the pipe HS when piston P is ascending. In order to make the flow continues during the upstroke an air chamber, such as that shown in Fig. 80, is always inserted between the valve a (Fig. 79) and the discharge point. As the water is forced violently into this chamber it compresses the confined air. It is, then, the reaction of this compressed air which is immediately responsible for the flow in the discharge tube, and as this reaction is continuous the flow is also continuos.

Fig. 81 represents one of the most familiar types of force pump, the double-acting steam fire engine. Let the student analyze the action of the pump from a study of the diagram.

4.3.8 The Cartesian diver

Descartes (1596-1650), the great French philosopher, invented an off device which illustrates at the same time the principle of the transmission of pressure by liquids, the principle of Archimedes, and the compressibility of gases. A hollow glass image in human shape [Fig. 82, (1)] has an opening in the lower end. It is partly filled with water and partly with air, so that it will just float. By pressing on the rubber diaphragm at the top of the vessel it may be made to sink or rise at will. Explain. If the diver is not available a small bottle of test tube [see Fig. 82, (2)] may be used instead. It works equally well, and brings out the principle even better.

4.3.9 The balloon

A reference to the proof of Archimeded's principle (Section 3.3.2, page 3.3.2) will show that it must apply as well to gases as to liquids. Hence any body immersed in air is bouyed up by a force which is equal to the weight of the displaced air. The body will therefore rise if its own weight is less than the weight of the air which it displaces.

A balloon is a large silk bag (Fig. 83) varnished so as to be air-tight and filled either with hydrogen or with common illuminating gas. The former gas weighs about $0.09\frac{\mathrm{kg}}{\mathrm{m}^3}$ and common illuminating gas weighs about $0.75\frac{\mathrm{kg}}{\mathrm{m}^3}$. It will be remembered that ordinary air weighs about $1.25\frac{\mathrm{kg}}{\mathrm{m}^3}$. It will be seen, therefore, that the lifting power of hydrogen per cubic meter, namely $1.20 - 0.90 = 1.11$ is more than twice the lifting power of illuminating gas, $1.20 - 0.75 = 0.45$. Nevertheless, on account of the comparative cheapness of the latter gas, its use is very much more common.

From the weights given about it is easy to calculate the ligting power of any balloon whose volume is known. Glasier and Coxwell's balloon had a volume of 90,000 ft^3, and was able to carry a load of about 600 lb.

Ordinarily a balloon is not completely illed at the start, for if it were, since the outside pressure is continually diminishing as it ascends, the pressure of the inside gas would subject the bag to enormous strain, and would surely burst it before it reached any considerable altitude. But if it is by partly inflated at the start, it can increase in volume as it ascends by simply inflating toa greater extent.

The parachute seen hanging from the side of the balloon in Fig. 83 is a huge umbrella-like affair, which after opening as in Fig. 84, descends very slowly on account of then enormous surface exposed to the air. The hole int he top allows air to escape slowly and thus keeps the parachute upright.

4.3.10 The divingbell

The diving bell (Fig. 85) is a heavy bell-shaped body with rigid walls, which sinks of its own weight. Formerly the workmen who went down in the bell had at their disposal only the amount of air confined within it, and the water rose to a certain height within the bell on account of the compression of the air. But in modern practice the air is forced in from the surface through a connecting tube (a, Fig. 86) by means of a force pump h. This arrangment, in addition to furnishing a continual supply of air, makes it possible to force water down to the level of the bottom of the bell. In practice a continual stream of bubbles is kept flowing out from the lower edge of the bell, as shown in Fig. 86.

The pressure of the air withing the bell must, of course, be the pressure existing within the water at the depth of the level of the water inside the bell, i.e. in Fig. 85 at the depth AC. Thus at a depth of 34 ft. the pressure is 2 atmospheres. Diving bells are used for putting in the foundations of bridge piers, doing subaqueous excavating, etc. The so-called *caisson*, much used in bridge building, is simply a huge stationary diving bell, which the workmen enter through compartments provided with air-tight doors. Air pumped into is precisely as in Fig. 86.

4.3.11 The diving suit

For most purposes, except those of heavy engineering, the diving suit has now replaced the diving bell. This suit is made of rubber with a metal helmet. The diver is sometimes connected with the surface by a tube (Fig. 87) through which air is forced down to him. It passes out into the water through the valve v in his suit. Buy more commonly the diver is entirely independent of the surface, carrying air under pressure of about 40 atmospheres in a tank on his back. This air is allowed to escape gradually through the suit and out into the water through the valve v as fast as the diver needs it. When he wishes to rise to the surface he simply admits enough air to his suit to make him float.

In all cases the diver is subjected to the pressure existing at the depth at shich the suit or bell communicates with the outside water. Divers seldom work at depths greater than 60 ft., and 80 ft. is usually considered the limit of safety. But in building the bridge over the Missippi at St. Louis, Missouri, the bells with their divers were sunk to a depth of 201 ft.

The diver experiences pain in the ears and above the eyes when he is ascending of descending, but not when at rest. This is because it requires some times for the air to penetrate into the interior cavities of the body and establish equal pressure in both directions.

4.3.12 The air brakes

Fig. 88 is a diagram which shows the essential features of the Westinghouse air brake. P is an air pipe leading to the engine, where a compression pump maintains air in the main cylinder under a pressure of about 70 lb. to the square inch. R is an auxiliary reservior which is placed under each car, and which connects with P through the triple valve V. So long as the pressure from the engine is on in P, the valve V is open in such a way that there is direct communication between P and R. But as soon as the pressure in P is diminished, wither by the engineer of by the accidental breaking of the hose coupling k, which connects P from car to car, the compressed air in R operates the velve V so as to shut off connection between R and the cylinder C. The piston H is thus driven powerfully to the left and sets the brake shoes against the wheels through the operation of levers attached to H. When it is desired to take off the brakes, pressure is again turned on in P. This operation opens V in such a way as to permit the compressed air in C to escape, and the spring S then pulls back the brake shoes from the wheels.

4.3.13 The bellows

Fig. 89 shows the construction of the ordinary blacksmith's bellows. Then the handle a rises and the point b in consequence falls, the vlave v opens and air from the outside enters the lower compartment C_1. When a is pulled down and b thus made to ascend, v at once closes, and as soon as the pressure with in C_1 has risen to the same value as that maintained by C_2 by the weights W, the valve

v' opens and air passes from C_1 to C_2. With this arrangement it will be seen that the current of air which issues from C_2 through the nozzle is continusous rather than intermittent, as it would be if there were no compartment and one valve.

4.3.14 The gas meter

The gas meter is a device which differs little in principle from the blacksmith's bellows. Gas from the city supply enters the mter through P (Fig. 90), and passes through the opening o into the compound compartment B of the meter. Here its pressure forces in the diaphragm d, at the same time forcing out the diaphragm d'. Each of these operations diminishes the size of the compartment A, for the diaphragm m is immovable. The gas already contained in A is therefore pushed out to the burners through the openings o' and e and the pipe p. As soon as compartment B is full, a lever which is worked by the movement of the diaphragms causes the slide value v to move to the left, thus closing o and shutting off connection between P and B, but at the same time opening o' and allowing the gas from P to enter compartment A through o'. The gas in B is now forced out through the openings o and e and the pipe p. The movement of the diaphragms is recorded by a clockwork device, the dials of which (Fig. 91) indicate the number of cubic feet of gas which have passed through the meter.

4.3.15 Questions and problems

Add this section

Chapter 5

Molecular Motions

5.1 Kinetic Theory of Gases

5.1.1 Molecular constitution of matter

In order to account for some of the simplest facts in nature,–e.g. the fact that two substances often apparently pssupy the same spact at the same time, as when two gases are crowded together in the same vessel, of when sugar is dissolved in water,–it is now universally assumed that all substances are composed of very minute particles called *molecules.* Spaces are supposed to exist between these molecules, so that when one gas enters a vessel which is already full of another gas, the molecules of the one scatter themselves about between the molecules of the other. Since molecules cannot be seen with the most powerful microscopes, it is evident that they must be very minute, and the number of them contained in a cubic centimeter of any substance must be enormous. Probably it would take as many as a thousand molecules laid side by side to make a speck long enough to be seen witht he best microscopes.

5.1.2 Evidence for molecular motions in gases

Certain very simple observations lead us to the conlclusion that the molecules of gasesk, even in a still room, must be in continual and quite rapid motion. Thus, if a little chlorine, or ammonia, of any gas of powerful odor is introduced into a room, in a very short time it will have become perceptible in all parts of the room. This shows clearly that enough of the molecules of the gas to affect the olfactory nerves must have found their way across the room.

Again, chemists tell us that if two globes, one containing hydroven and the other carbon dioxide gas, be connected as in Fig. 93 and the stopcock between them opened, after a few hours chemical analysis will show that each of the globes contains the two gases which is at first sight very suprising, since carbon dioxide gas is about twenty-two times as heavy as hydrogen. This mixing of gases in apparent violation of the laws of weight is called *diffusion*.

We see then that such simple facts as the transference of odors and the diffusion of gases furnish very convincing evidence that the molecules of a gas are not as rest, but are continually moving about.

5.1.3 Molecualr motions and the indefinite expansibility of a gas

Perhaps the most striking property which we have found gassed to possess is the property of indefinite or unlimited expansibility. The existence of this property was demonstrated by the fact that we were able to obtain a high degree of exhaustion by means of an air pump. No matter how much air was removed from the bel ljar, the remainder at once expanded and filled the entire vessel. In fact, it was only because of this property that the air pump was able to perform its functions at all.

In order to explain these facts it used to be assumed that the molecules of gases exert mutual repulsion upon one another. This theory has now, however, been completely abandoned, for it has been conclusively shown that no such repulsions exist. The motions of the molecules alone furnish a thoroughly satisfactory explanation of the phenomenon. As soon as the piston of the air pump is drawn up, some of the molecules follow it because they were already moving in that direction, and not on account of any repulsion exerted upon them by the molecules below. The phenomenon is precisely the same as that illustrated in Fig. 93 where the carbon dioxide molecules moved up into the globe containing hydrogen; only in the latter case the operation took much more time because the upward motion of the carbonic acid molecules was hindered by collisions with the hydrogen molecules.

The fact that, however, rapidly the piston of the air pump is drawn up, gas always appears to follow it instantly, leads us to the conclusion that the natural velocity possessed bythe molecules of gases must be very considerable.

5.1.4 Molecular motions and gas pressures

if the molecules of gases do not repel one another, how are we to account for the fact that gases exrt such pressures as they do against the walls of vessels which sontain them? We have found that in an ordinary room the air presses against the walls with a force of 15 lb. to the square inch. Within an automobile tire this pressure may amount to as much as 100 lb., and the steam pressure within the boiler of an engine is often as high as 240 lb. per square inch. Yet in all these cases we may be certain that the molecules of the gas are separated from eachother by distances which are largh in comparison with the diameters of the molecules; for when we reduce steam to water it shrinks to $\frac{1}{1600}$ of its original volume, and when we reduce air to the liquid form it shrinks to about $\frac{1}{800}$ of its ordinary volume.

The explanation is at once apparent when we reflect upon the *motions* of the molecules. For just as a stream of water particles from a hose exerts a continuous force against a wal on which it strikesk, so the blows which the innumerable molecules of a gas strike against the walls of the containing vessels containing only gas–balloons, for example,–do not collapse under the enourmous ecternal pressures to which we know them to be subject to.

5.1.5 Explanation of Boyle's law

It will be remembered that it was discovered in the last chapter that when the density of a gas is doubled, the tempraturt remaining constant, the pressure is found to double also. When the density was trebled, the pressure which a gas exerts against a given surface is due to blows struck by an enormous number of swiftly moving molecules; for doubling the number of molecules in the given space–i.e. doubling the density–would simply double the number of blows struck per second against that surface, and hence would double the pressure. While the kinetic theory of gases which is here presented accounts in this simple way for Boyle's law, the theory of molecular repulsions cannot be reconciled with it.

5.1.6 Moleculat velocities

From the weight of a cubic centimeter of air under normal conditions, and the known force whish it exerts per square centimeter,–viz. 1033g.,–it is possible to calculate the velocity which its molecules must possess in order that they may produce by their collisions against the walls this amount of force. Further, since a cubic centimeter of hydrogen which is in condition to exert the same pressure as a cubic centimeter of air weighs only one fourteenth as much as air, it is evident that the hydrogen molecules must be moving much more raplidly than the air molecules, or else they could not exert the same pressure. The result of the calculation gives to the air molecules under normal conditions a velocity of about 445 $\frac{\mathrm{m}}{\mathrm{s}}$. The speed of a cannon ball is seldom greater than 800 $\frac{\mathrm{m}}{\mathrm{s}}$. It is easy to see then, since the molecules of gases are endowed with such speeds, why, air, for example, expands instantly into the space left behind by the rising piston of the air pump, and why any gas always fills completely the vessel which contains it.

5.1.7 Diffusion of gases through porous walls

Strong evidence for the cottectness of the above views is furnished by the following experiment.

let a porous cup of unglazed earthenware be closed with a rubber stopper through whish a glass tube passes, as in Fig. 94. Let the tube be dipped into a dish of colored water, and a jar contaiing hydrogen placed over the porous cup, or let the jar simply be held in the position shown in the figure, and illuminating gas passed into it by means of a rubber tube connected with a gas jet. The rapis passage of bubbles out through the water will show that the gaseous pressure inside the sup is rapidly increading. Now let the bell jar be lifted, so that the hydrogen is removed from the outside. Water will at once begin to rise in the tube, showing that the inside pressure is now rapidly decreasing.

The explanation is as follows. We have learned that the molecules of hydrogen have about four times the velocity of th molecules or air. Hence, if there are as many hydrogen molecules per cubic centimeter outside the cup as there are air molecules per cubic centimeter inside, the hydrogen molecules will strike the outside of the wall four times as frequently as the air molecules will strike the inside. Hence, in a given time, the number of hydrogen molecules which pass into the interior of the cup through the little holes in the porous material will be four times as great as the number of air particles which pass out. Since the inside is thus gaining molecules faster than it is losing them, and since the pressure of a gas at a given temperature is determined solely by the number of molecules which are bombarding the wall, the inside pressure mush increase until the number per cubic centimeter inside is so much larger than the number outside that molecules pass out as fast as they pass in. When the bell jar is removed the hydrogen which has passed inside now begins to pass out faster than the outside air passes in, and hence the inside pressure is diminished.

5.1.8 Temperature and molecular velocity

mperature_and_molecular_velocity

The effects whish are ovserved when a gas is heated furnish further evidence that its molecules are in motion.

Let a bulb of air B be connected with a water manometer m, as in Fig, 95. If the bulb is warmed by holding a Bunsen burner beneath it, or even by placing the hand upon it, the water at m will at once begin to descend, showing that the pressure exerted by the air contained in the bulb has been increased in its temperature. If B is cooled with ice of ether the water will rise at m.

Now if gas pressure is due to the bombardment of the walls by the molecules of the gas, since the number of molecules in the bulb can scarcely have been changed by slightly heating it, we are forced to conclude that the increase in pressure is due to an increase in the *velocity* of the molecules whish are already there. The temperature of a given gas, then, from the standpoint of the kinetic theory, is determined simply by the mean velodity of the gas molecules. To increase the temperature is to increase the average velocity of the molecules, and to diminish the temperature is to diminish this average molecular velocity. The theory thus furnishes a very simple and natural explanation of the fact of the exxpansion of gases with a rise in temperature.

5.1.9 Questions and problems

Add this section

5.2 Molecular Motions In Liquids

5.2.1 Molecular motions in liquids and evaporation

Evidence that the molecules of liquids as well as those of gases are in a state of perpetual motion is found, first, in the familiar facts of evaporation. We know that the molecules of a liquid in an open vessel are continually passing off into the space above; for it is only a matter of time when the liquid completely disappears and the vessel becomes fry. Now it is hard to imagine a way in which the molecules of a liquid, while in the liquid condition, are in motion. As soon, however, as such a motion is assumed, the facts of evaporation become perfectly intelligible. For it is to be expected that in the jostlings and collisions of rapidly moving liquid molecules an occasional molecule will acquire a velocity much greater than the a verage. This molecule then, because of the unusual speed of its motion, break away from the attraction of its neighbors and fly off into the space aboce. This is indeed the mechanish by which we now bwlieve the the process of evaporation goes on.

5.2.2 Molecular motions and the diffusion of liquids

One of the most convincing arguments for the motions of molecules in gases was found in the fact of diffusion. But precisely the same sort of phenomena are observable in liquids.

Thus, let a jar be partially fille dwith water colored with blue litmus, and let a little sulphuric acid be carefully introduced into the bottom of the jar, beneath the water, by means of a thistle tube (Fig. 96). Whenever acit comes in contact with blue litmus it turns it red. Since the sulphuric is 1.8 times as heavy as water, it as first remains at the bottom, and the line of separation between it and the water will befound to be fairly sharp; but in the course of a few hours, even though the jar is kepr perfectly quiet, the red color will be found to have spread considerably toware the top of the jar, showing that the acid molecules have gradually found their way toward the top.

Certainly, then, the molecules of a liquid must be endowed with the power of independent motion.

5.2.3 Molecular motions and the expansion of liquids

_expansion_of_liquids

The fact of the expansion of gases with a rise of temperature was looked upon as evidence that the molecules of gases are in motion, the velocity of this motion increasing with an increase in temperature. But precisely the same property belongs to liquids also.

Thus, let the bulb (Fig. 97) be heated with a Bunsen burner. The contained liquid will be found to expand and rise in the tube.

It is natural io infer that the cause of this increase in volume is the same as before; i.e. the velocity of the molecules of the liquid has been increased by the rise in temperature, and they have jostled one another farther apart, and tues caused the whole volume to be enlarged. According to this view, then, an increase in temperature in a liquid, as in a gas, means an increase in the mean velocity of the molecules, and conversely a decrease in tempearature means a decrease in this average velocity.

5.2.4 Evaporation and temperature

If it is true that increase in temperature means increase in the mean velocity of molecular motion, then the number of molecules which chance in a given time to acquire the velocity necessary to carry them into the space above the liquid, ought to increase as the temperature increases; i.e. evaporation ought to take place more rapidly at high temperatures than at low. Common observation teaches that this is true. Damp clothes become dry under a hot flatiron but not under a cold ont; the sidewalk dries more readily in the sun thatn in the shade; we put wet objects near a hot stove of radiator when we wish them to dry quickly.

5.3 Properties of Vapors

5.3.1 Saturated vapor

If a liquid is placed in an open vessel, there ought to be no limit to the number of molecules which can be lost by evaporation, for as fast as the molecules emerge from the liquid they are carried away by the air currents. As a matter of fact, experience teaches that water left in an open dish does waste away until the dish is completly dry.

But suppose that the liquid is evaporating into a closed space, such as that shoen in Fig. 98. Since the molecules which leave the liquid cannon escape fromt he space S, it is clear that as time goes on the number of molecules which have passed off from the liquid into this space must continually increase; in other words, the density of the vapor in S must frow greater and greater. The question which at once suggests itself is, "Is there any limit to the density of which this vapor can attain?" i.e. "Will evaporation fo on indefinitely into the space S, so that the vessel of liiquid placed in it will ultimately dry up?" Experiment has very positively answered this question in the negative. A vessel of water placed in an air-tight bell har will never waste away. Hence there must be a limit to the possible amount of evaporation into a closed space above a liquid, i.e. to the density which the vapor can attain. When this limit is reached the vapor is said to be *saturated*.

5.3.2 Explanation of saturation

The kinetic theory furnishes a very simple explanation of the facts of saturation. The molecules which have escaped into S (Fig. 98) are moving about in all directions within this space. Whenever one of them in its motions chances to strike the surface of the liquid, it reënters and does not again escape unless it chances to acquire again the velocity which is necessary for the escape of *any* molecule from the liquid. It is clear that the more molecules there are present in the space above the liquid, the more frequently will some of the strike the surface of the liquid and return to it permanently in the manner just described. In fact, if we double the number of molecules in the space S, we must double the numbe which strike the surface of the liquid per second, and hence double the number which will return to the liquid per second. Evidently, then, as the natural process of evaporation causes the vapor to become more and more dense in S, a condition must soon be reached when the number of molecules which return per second from the vapor to the liquid is equal to the number which pass out of the liquid pre second into the space S; for the number which pass out of the liquid per second depends simply upon how many acquire the velocity necessary for escape, and has nothing to do with the amount of vapor abot the liquid. When this condition of saturarion has been reached there will be a containual exchange of molecules between the liquid and the vapor; but he liquid will no longer wasted away and the vapor will no longer increase in density. The vapor is then in the *saturated* condition.

5.3.3 Pressure of a saturated vapor

We have learned that any gas of vapor presses out against the walls of the containing vessel because the blows which its moving molecules strikee against these walls. We have learned also from Boyle's law that the pressure which a gas or vapor exerts is directly proportional to its dcdensity, i.e. to the number per cubic centimeter to strike such blows. The pressure which the vapor in the space S ecerts against the walls of S must therefore increase in just the proportion in which the density of the vapor increases, and reach a maximum when the density reaches a maximum. This maximum pressure which a vapor can exert at a given temperature is called *the pressure of the saturated vapor.*

5.3.4 Measurement of the pressure of a saturated vapor

_of_a_saturated_vapor

Let four Torricellian tubes be set up as in Fig. 99, and with the air of a curved pipette (Fig. 99) let a drop of ether be introduced into the bottom of tube 1. This drop will at once rise to the top and a portion of it will evaporate into the vacuum which exists above the mercury. The pressure of this vapor will push down the mercury column, and the number of centimeters of this depression will of course be measured of the pressure of the vapor. It will be observed that the mercury will fall almost instantly to the lowest level which it will ever reach,– a fact which indicates that it takes but avery short time for the condition of saturation to be attained. In the same way let alcohol and water be introduced into tubes 2 and 3 respectively.

While the pressure of the saturated ether vapor at the temperature of the room will be found to be as much as 40 cm., that of alcohol will be found to be but 4 or 5 cm., and that of water only 1 or 2 cm.

5.3.5 No change in the volume of a saturated vapor can affect its density or pressure

Suppose that after the condition of saturation has been reached in the space S (Fig. 98)–i.e. after the number of molecules which return from the vapor to the liquid per second has become equal to the number whish pass from the liquid to the vapor per second–the volume of the space S were to be suddenly decreased so as to increase momentarily the number of molecules per cubic centimeter in the space above the liquid. This would increase the number of vapor molecules which strike the liquid surface per second, and thus increase the rate aet which molecules return to the liquid without changing inn any way their rate of emergence. Hence the vapor would necessarily grow less and less dense because of this uncompensated loss of molecules, until the number entering per second was again reduced to the number emergin per second,–i.e. until the vapor density in S became the same as at the first. We conclude, then, that the density of a vapor in contact with its liquid cannot be permanently increased by compressing it so long as the temperature remains the same.

If, on the other hand, the density of the vapor above the liquid is momentarily diminished by suddenly increasing the volume of the space S, more molecules will emerge per second from the liquid than enter it from the vapor. Consequently the density of the vapor must increase until it reaches the old equilibrium value. In a word, then, if we decrease the volume of a saturated vapor, it should condense until the former density is restored; and if we increase the volume, more liquid should evaporate until the first condition is again regained. In order to verify this conclusion let the following striking experiment be performed.

Let two Torricelli tubes be placed in a long cistern of mercury, as in Fig. 100, and let a drop of ether be admitted into one, while enough air is allowed to pass into the other to reduce the mercury height to about the same level in the two tubes. let the tubes be pushed down into the cistern so as to diminish the volume of the gases in the upper part. In the air tube this operation will be found to decrease the height of the mercury column db, showing that the pressure of the air within the tube has been increased, as of course it ought to be in accordance with Boyle's law, the volume having been diminished. But in the ether tube the height ab will be found to have been only momentarily changed by either lowering or raising the tube, thus showing that the pressure, and therefore the density, of the vapor remains constant for all changes in volume. An increase in the volume simply causes more of the liquid to evaporate, while a decrease causes some of the vapor to condense.[1]

5.3.6 Influence of temperature on the density and pressure saturated vapor

ity_and_pressure_saturated_vapor

Let a Bunsen flame be passed quickly to and fro across the tubes of Fig. 100, near the upper level of mercury. The heights ab and db will fall in both, but the fall will be found to be much greater in the ether tube than in the air tube.

[1]if enough mercury is not at hand to perform the experiment as indicated in Fig. 100, this property of the saturated vapor may be illustrated almose as wel by simply inclining the vapor tubes of Fig. 99. This will decrease the volume, but the upper level of the mercury will remain at the same distance above the table, showing that the pressure has undergone no change.

Since the two tubes have been about equally heated, there must have been about the same relative increase in molecular velocity in each. Hence the excess of pressure which the heating has produced in the ether tube must be due to increased evaporation, i.e. to an increase in the number of molecules per cubic centimeter in the ether vapor.

The experiment proves that both the pressure and the density of a saturated vapor increase rapidly with the temperature. This was to have been expected from our theory; for increasing the temperature of liquids increases the mean velocity of its molecules and hence increases the mean velocity of its molecules and hence increases the number which attain each second the velocity necessary for escape.

Let a piece of ice be held about the tubes near the top of the mercury. The mercury will rise in both, but much more rapidly in the ether tube than in the air tube, thus showing that the ether vapor is condensing.

The experiment shows that if the temperature of the saturated vapor is diminished, it condenses until its density is reduced to that corresponding to saturation at the lower temperature. How rapidly the density and pressure of saturation increase with temperature may be seen from the following graph of collected data.[2]

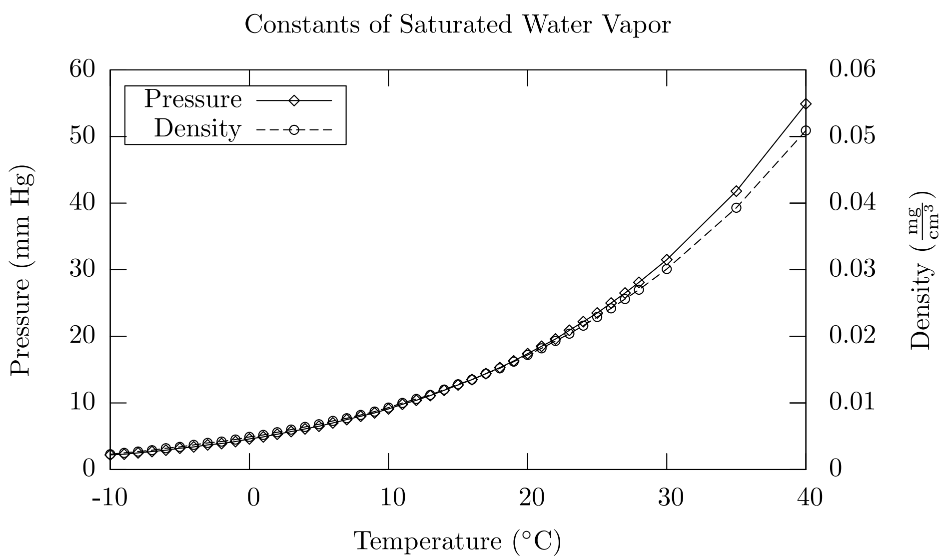

Figure 5.1: Constants of Saturated Water Vapor

Constants_of_Saturated_Water_Vap

[2]The data for this graph may be found in the *Introductory Physics* source files in "plots/Constants_of_Saturated_Water_Vapor.dat".

5.3.7 The influence of air on evaporation

_influence_of_air_on_evaporation

We observed that when a drop of ether was inserted into the Torricellian tube the mercury fell *very suddenly* to its final position, showing that in a vaccum the condition of saturation is reached almost instantly. This was to have been expected from the great velocities which we found the molecules of gases and vapors to possess.

In order to see what effect the presence of air has upon the vaporization, let a drop of ether be introduced into the air tube of the last experiment (Fig. 100). The mercury will not be found to sink instantly to its final level as it didd before, but although it will fall rapicly at first, it will continue to fall slowly for several hours. At the end of a day, if the temperature has remained constant, it will show a depression which indicates a vapor pressure of the ether just as great as that existing in a tube which contains air.

The experiment leads, then, to the rather remarkable conclusion that *just as much liquid will evaporate into a space which is already full of air as into a vacuum.* The air has no effect except to retard greatly the *rate of evaporation.*

5.3.8 Explanation of the retarding influence of air on evaporation

This retarding influence of air on evaporation is easily explained by the kinetic theory; for while in a vacuum the molecules which emerge from the surface fly at once to the top of the vessel, when air is present the escaping molecules collide with the air molecules before they have gone any appreciable distance away from the surface (probably less that 0.00001 cm.), and only work their way up to the top after an almost infinite numver of collisions. Thus, while the space immediately above the liquid may become saturated very quickly, it requires a long time for the condition of saturation to reach the top of the vessel. That ultimately, however, as much liquid will evaporate into a space containing air as into a vacuum is to be expected from the fact that evaporation ceases only when as many molecules of the liquid substance return to the liquid per second as escape per second. This number which returns depends simply on the number of molecules of the liquid which are present per cubic centimeter in the space above, and not at all on how many molecules of other gases may be present there.

It must not be forgotten, however, that at a given temperature the *pressure* existing within a vessel containing gases is simply due to the total number of molecules per cubic centimeter which are striking blows against each square centimeter of the wall. Therefore, when a liquid evaporates into a closed vessel already containing air, the pressure gradually increases, and is *ultimately equal to the air pressure plus the pressure of the saturated vapor.* When a liquid evaporates in an open vessel,–i.e. under constant pressure,–its molecules crowd an equal number of molecules of air.

5.3.9 Questions and problems

Add this section

5.4 Hygrometry

5.4.1 Condensation of water vapor from the air

Were it not for the retarding influence of air upon evaporation we should be obliged to live in an atmosphere which would be always completely saturated with water vapor; for the evaporation from oceans, lakes, and rivers would almost instantly saturate all the regions of earth. This condition–one in which moist clothes would never dry, and in which all objects would be perpetually soaked in moisture–would exceedingly uncomfortable, if not altogether unendurable.

But on account of the slowness with which, as the last experiment showed, evaporation takes place into air, the water vapor which always exists in the atmosphere is usually far from saturated, even in the immediate neighborhood of lakes and rivers. Since, however, the amount of vapor whish ic necessary to produce saturation rapidly decreases with a fall in temperature, if the temperature decreases continually in some unsaturated locality, it is clear that a point must soon be reached at which the amount of vapor already existing in a cubic centimeter of the atmosphere is the amount corresponding to saturation. Then, Influence_of_temperature_on_the_density_and_pressure_saturated_vapor in accordance with the facts discovered in section 5.3.6, if the temperature still continues to fall, the vapor must begin to condense. Whether it condenses as dew, or cloud, or fog, or rain will depend upon how and where the cooling takes place.

5.4.2 The formation of dew

If the cooling is due to the natural radiation of heat from the earth at night after the sun's warmh is withdrawn, the atmosphere itself does not fall in temperature nearly as rapidly as do solid objects on the earth, such as blades of grass, trees, stones, etc. The layers of air which come into immediate contact with these cooled bodies are themselves cooled, and as they thus reach a temperature at a saturated condition, they begin to deposit this moisture, in the form of dew, upon the cold objects. The drops of moisture which collect on an ice pitcher in summer illustrate perfectly the whole process.

5.4.3 The formation of fog

If the cooling at night is so great as not only to bring the grass and trees below the temperature at which the vapor in the air in contact withthem is in a state of saturation, but also to lower the whole body of air near the earth below this temperature, then the condensation takes place not only on thesolid objects but also on dust particles suspended in the atmosphere. This constitutes fog.

Cloud
Rain
Hail
Snow
Dew-point

5.4.4 The formation of clouds, rain, hail, and snow

When the cooling of the temosphere takes place at some didtance *above* the earth's surface, as when a warm current of air enters a cold region, if the resultant temperature is below that at which the amount of mositure already in the air is sufficient to produce saturation, this excessive mositure immeciately condenses about floating dust particles and forms a *cloud*. If the cooling is usfficient to free a considerable amouunt of mositure, the frops vecome large and fall as *rain*. If the falling rain passes through cold regions, it freezes into *hail*. If the temperature at which condensation begins is below freezing, the condensing moisture forms into *snowflakes*.

5.4.5 The dew-point

The temperature to which the atmosphere must be cooled in order that condensation may begin is called the *dew-point*. This temperature may be found by partly filling with water a brightly polished vessel of 200 or 300 cm^3 capacity and dropping into it little pieces of ice, stirring thoroughly at the same time with a thermometer. The dew-point is the temperature iindicated by the thermometer at the instant a film of moisture appears upon the polished surface. In winter the dew point is usually below freezing, and it will therefore be necessary to ass salt to the ice and water in order to make the film appear. The experiment may be performed equally well by bubbling a current of ait through ether contained in a polished tube (Fig. 101).

5.4.6 Humidity of the atmosphere

From the dew-point and the table given in section 5.3.6 page 62, we can easily find what is commonly known as the *relative humidity*, of the *degree of saturation* of the atmosphere. This quantity is defined as *the ratio between the amount of moisture actually present in the air per cubic centimeter, and the amount which would be present if the air were completely saturated*. This is precisely the same as the ratio between the pressure which the water vapor present in the air exerts, and the pressure which it would exert if it were present in sufficient quanitty to be in the saturated condition. As example will make clear the method of finding the relative humidity.

Suppose that the dew-point were found to be 15°C on a day on which the temperature of the room was 25°C. The amount of mositure actually present in the air then saturates is at 15°C. We see from the Pressure axis on the graph that the pressure of saturated vapor at 15°C is 12.7 mm of mercury. This is then the pressure exerted by the vapor in the air at the time of our experiment. Running aross the graph, we see that the amount of moisture required to produce saturation at the temperature of the room, i.e. at 25°C, would exert a pressure of 23.5 mm of mercury. hence at the time of the experiment the air contains $\frac{12.7}{23.5}$, of 0.54, as much water vapor as it might hold. We say, therefore, that the air is 54% saturated, of that the relative humidity is 54%.

5.4.7 Practical value of humidity determinations

From humidity determinations it is possible to obtain much information regarding the likelihood of iran of frost. Such observations are continually made for this purpose at all meteorogical stations. Further, they are made in greenhouses to see that the air does not become too dry for the welfare of the plants, and also in hospitals and public buildings, and veen in privite dwelling, in order to insure the maintenance of hygenic living conditions. For the most healthful conditions the relative humidity should be kept at from 50% to 60%.

5.4.8 Cooling effect of evaporation

let three shallow dishes be partly filled, the first with water, the second with alcohol, and the third with ether, the bottle from which these liquids are obtained having stood in the room long enough to acquire its temperature. let three students carefully read as many thermometers, first before their bulbe have been immersed in the respectice liquids and then after. In every case the temperature of the liquid in the shallow vessel with be found to be somewhat lower than the temperature of the air, the difference being greatest in the case of the ether and lease in the case of water.

It appears from this experiment that an evaporating liquid assumes a temperature somewhat lower than its surroundings, and that the substances which evaporate the mose readily, i.e. those which have the greatest vapor pressures at a given temperature (see section 5.3.4, assume the lowest temperatures.

Another way of establishing the same truth is to place a few drops of each of the above liquids in succession on the bulb of the arrangement shown in Fig. 95, and observe the rise of water in the steam; or, more simply still, to place a few drops of each liquid on the back of the hand, and notice that the order in which they evaporate–namely, ether, alcohol, water–is the order of gtearest cooling.

5.4.9 Explanation of the cooling effect

The kinetic theory furnishes a simple explanation of the cooling effects of evaporation. We saw that in accordance with this theory evaporation means an escape from the surface of those molecules which have acquired velocities considerably above the average. But such a continual loss from a liquis of its most rapidly moving molecules involves, of course, a continual diminishing of the average velocity of the molecules left behind in the liquid state, and this means a decrease in the temperature of the liquid (see sections 5.1.8 and 5.2.3).

Again, we should expect the amount of cooling to be proportional to the rate at which the liquid is losing molecules. Hence, of the three liquids studied, ether should cool most rapidly, since it shows the highest vapor pressure at a given temperature and therefore the highest rate of emission of molecules. The alcohol shoulc some next in order, and the water last, as was in fact observed.

5.4.10 Freezing by evaporation

The_influence_of_air_on_evaporation

In section 5.3.7 it was shown that a liquid will evaporate much more quickly into a vacuum than into a space comtaining air. Hence if we place a liquid under the reciever of an air pump and exhaust rapidly, we ought to expect a much greater fall in temperature than when the liquid evaporates into air. This conclusion may be strikingly verified as follows.

Let a thin watch cladd be filled with ether and plced upon a drop of cold water, preferably ice water, which rests upon a thin glass plate. Let the whold arrangement be placed underneath the reciever of an air pump and the air rapidly exhausted. After a few minutes of pumping the watch glass will be found frozen to the plate.

It was by evaporating liquid hydrogen in this way that Professor james Dewar of London, in 1900, attained the lowest temperature which has ever been reached, viz. $-260°$C.

5.4.11 Effect of air currents upon evaporation

Let four thermometer bulbs, the first of which is dry, the scond wetted with water, the third with alcohol, and fourth with ether, be rapidly fanned and their respective temperatures observed. The results will show that in all of the wetter thermometers the fanning will considerably augment the cooling, but the dry thermometer will be wholly unaffected.

The reason that fanning it removes evaporation, and therefore cooling, is that it removes the saturated layers of vapor which are in immediate contact with the liquid and replaces them by unsaturated layers into which new evaporation may at once take place. From the behavior of the dry-bulb thermometer, whoever, it will be seen that fanning produces cooling only when it can thus hasten evaporation. A dry body at the temperature of the room is not cooled in the slightest degree by blowing a current of air arcoss it.

5.4.12 The wet-and-dry-bulb hygrometer

In the wet-and-dry-bulb hygrometer (Fig. 102) the principle of cooling by evaporation finds a very useful application. This instrument consists of two thermometers, the bulb of one of which is dry, while that of the other is kept continually moist by a wick dipping into a vessel of water. Useless the air is saturated the wet bulb indicates a lower temperature than the dry one, for the reason that evaporation is continually taking place from its surface. How much lawer will depend on how rapidly the evaporation proceeds; and this in turn will depend upon the relative humidity of the atmosphere. Thus in a completely saturated atmosphere no evaporation whatever takes place at the wet bulb, and it consequently indicates the same temperature as the dry on. By comparing the indications of this instrument with those of the dwq-point hygrometer (Fig. 101), tables have been constructed which enable one to determine at once from the readings of the two thermometers both the relative humidity and the dew-point. On account of their convenidnce instruments of this sort are used almost exclusively in pratical work. They are not very reliable unless the air is made to circulate about the wet bulb before the reading is taken.

5.4.13 Effect of increased surface upon evaporation

Let a small tube containing water be dipped into a larger tube, or a small glass, containing ether, as in Fig. 103, and let a current of air be forced rapidly through the ether with an aspirator. The water within the test tube will be frozen in a few minutes.

The effect of passing bubbles through the ether is simply to increase enormously the evaporating surface, for the ether molecules which could before escape only at the upper surface can now excape into the air bubbles as well.

5.4.14 Factors affecting evaporation

The above results may be summarized as follows: The rate of evaporation depends (1) on the nature of evaporation liquid; (2) on the temperature of the evaporating liquid; (3) on the humidity of the space into which the evaporation takes plce; (4) on the density of the air or other gas above the evaporating surface; (5) on the rapidity of the circulation of the air above the evaporating surface; (6) on the extent of the exposed surface of the liquid.

5.5 Molecular Motions In Solids

5.5.1 Evidence for molecular motion of solids

We have inferred that the molecules of liquids are in motion, in part at least, from the fact that liquids increase in volume when the temperature is raised, and from the fact that molceules of the liquid can usually be detected in a gaseous condition above the surface. Both of these reasons apply just as well in the case of solids.

Thus the facts of ecpansion of solids with an increse in temperature may be seen on every sidee. Railroad tails are laid with spaces between their ends so that they may expand during the heat of summer without crowding each other out of place. Wagon tires are made smaller than the wheels which they are to fit. They are then heated until they become large enough to be driven on, and in cooling they shrink again and thus grip the wheels with immense force. A common lectureroom demonstration of expansion is the following.

Let the ball B, which when cool slips through the ring R, be heated in a Bunesn flame. It will now be found too large to pass through the ring; but if the ring is heated, ot if the ball is again cooled, it will pass through easily (see Fig. 104).

Sublimation

5.5.2 Evaporation of solids

That the molecules of a solid substance are found in a vaporous condition above the surface of the solids, as well as above that of a liquid, is proven by the often observed fact that ise and snow evaporate wven though they are kept constantly below the freezing point. Thus wet clothes dry in winter after freezing. An even better proof is the fact that the odor of camphor can be detected many feet away from the camphor crystals. The evaporation of solids may be rendered visible by the following striking experiment.

Let a few crystals of iodine be placed on a watch glass and heated gently with a Bunsen flame. The visible vapor of iodine will appear above the crystals though none of the liquid is formed. A great many substances at high temperatures pass thus from the solid to the gaseous condition without passing through the liquid stage at all. This process is called *sublimation*.

5.5.3 Diffusion of solids

it has recently been demonstrated that if a layer of lead is placed upon a layer of gold, molecules of gold may in time be detected throughout the whole mass of lead. This difusion of solids into one another at ordinary temperatures has been shown only for these two metals, but at higher temperatures, e.g. 500°C., all of the metals show the same characteristics to quite a surprising degree.

The evidence for the existence of molecular motion in solids is then no less strong than in the case of liquids.

5.5.4 The three states of matter

Although it has been shown that in accordance with surrent belief the molecules of all substances are in very rapid motion, and that the temperature of a given substance, whether in the solid, liquid, of gaseous condition, is determined by the average velocity of its molecules, yet differences exist in the kind of motion which the molecules in the three states possess. Thus in the solid state it is probable that the molecules oscillate with great rapidity about certain fixed points, always being held by the attractions of their neighbors, i.e. the *cohexive forces* (see section Cohesion_and_adhesion ??), in practically the same positions with reference to other molecules in the body. In rare instances, however, as the facts of diffusion show, a molecule breaks away from its constraints. In liquids, on the other hand, while the molecules are, in general, as close together as in solids, they slip about with perfect ease over one another and thus have no fixed positions. This assumption is necessitated by the fact that liquids adjust themeslves readily to the shape of the containing vessel. In gases the molecules are comparatively far apart, as is evident from the fact that a cubic centimeter of water occupies about 1600 cm^3. when it is transformed into steam; and furthermore they exert practically no cohesive force upon one another, as is shown by the indefinite expansibility of gases.

5.5.5 Questions and problems

Add this section

Chapter 6

Molecular Forces

[1]

6.1 Molecular Forces In Solids. Elasticity

6.1.1 Proof of the existence of molecular forces in solids

The fact that a gas will expand without limit as the volume of the containing vessel is increased seems to show very conclusively that the molecules of gases do not exert any appreciable attractive forces upon one another. In fact, all of the experiments of the last chapter upon gases showed that such substances certainly behave as they would if they consisted of *independent molecules* moving hither and thither with great velocities and influencing each other's motions only at the instances of collision. Between collisions the molecules doubeless move in straight lines. It must not, however, be thought that the distances moved by a single bolecule between successive collisions are large. In ordinary air these distances probably do not average more than 0.00009 mm. Small, however, as this distance is, it is at least one hundred times the diameter of a molecule.

But that the molecules of *solids*, on the other hand, cling together with forces of great magnitude is proven by some of the simplest facts of nature; for solids not only do not expand like gases, but it often requires enormous forces to pull their molecules apart. Thus a rod of cast steel 1 cm. in diameter may be loaded with a weight of 8.8 tons before it will be pulled in two.

6.1.2 Tensile strengths

In order to compare the strengths of the forces which hold together the molecules of different substances, let three wires, all of the same diameter, e.g. 0.25 mm. (number 30), but consisting of three different materials, such as steel, brass, and aluminum, be wrapped side by side about a cylindrical rod, as in Fig. 105, and

[1] Add in the footnote from page 107 of A First Course In Physics.

Elasticity
Elasticity

weights added successively to the wires until they break. The breaking weights will be found to differ greatly for the three wires.

Tests made by methods similar to the above show that the *tensile strengths* of wires of the same material are directly proportional to the cross sections. This was to have been expected, since doubling the cross section doubles the number of molecules which must be pulled apart. The following are the weights in kilograms necessary to break drawn wires of different materials, 1 mm^2 in cross section.

Table 6.1: Maximum Tensile Strengths for Wires of 1 mm^2 Cross Section

Maximum_Tensile_Strengths

Substance	Force (N)	Substance	Force (N)
Steel	893	Platinum	422
Iron	755	Silver	363
Copper	500	Lead	26

6.1.3 Elasticity

Elasticity

We can obtain additional information about the molecular forces existing in different substances by studying what happens when weights applied are not large enough to break the wires.

Thus let a long steel wire, e.g. number 26 piano wire, be suspended from a hook in the cieling, and let the lower end be wrapped tightly about one end of a meter stick, as in Fig. 106. Let the fulcrum c be placed in a notch in the stick at a distance of about 5 cm. fro mthe point of attachment to the wire, and let the other end of the stick be provided with a knitting needle, one end of which is opposite the vertical mirror scale S. Let enough weights be applied to the pan P to place the wire under slight tension; then let the reading of the pointer p on the scale S be taken. Let three or four kilogram masses be added successively to the pan and the corresponding positions of the pointer read. Then let the readings be taken again as the weights are successively removed. In the last operation the pointer will probably be found to some back exactly to its first position.

This characteristic which the steel has shown in this experiment, of returning to its original length when the stretching weights are removed, is an illustration of a property possessed to a greater or less extent by all solid bodies. It is called *elasticity*.

6.1.4 The measure of elasticity

The_measure_of_elasticity

The relative amounts of elasticity possessed by different substances are found by subjecting wires of exactly the same dimensions, but of different materials, to tests like that used on the steel wire above. One substance is siad to have twice as high an elastic constant, or simply to be twice as *elastic* as another, when it requires twice as much force to procuce the same stretch; of, to state the same thing in a slightly different way, when with the same stretch it tends to spring back with twice the force. Thus it was found that if it required 20 kg. to stretch a given steel wire through 1 mm., it will require but 12 kg. to stretch an exactly similar copper wire through 1 mm., and 6 kg. to produce the same stretch in a similar wire of aluminum. Steel is therefore about 1.7 times as elastic as copperand 3.3 times as elastic as aluminum. It will be seen that when elasticity is measured in this way India rubber has a very small elastic constant, for it requires only a very small force to produce a considerable stretch. Elastic

6.1.5 Limits of perfect elasticity

If a sufficiently large weight is applied to the end of the wire of Fig. 106, it will be found that the pointer does not return exactly to its original position when the weight is removed. We say, therefore, that steel is *perfectly elastic* only so long as the distorting forces are kept within certain limits, and that, as soon as these limits are overstepped, it no longer shows perfect elasticity. Different substances differ very greatly in the amount of distortion which they can sustain before they show this failure to return completely to the original shape. Thus a drawn copper wire 1 mm. in diameter shows perfect elasticity until the stretching force exceeds about 12 kg., while a similar steel wire returen completely to its original length so long as the stretching force is less than 42 kg. Since, according to the results of section The_measure_of_elasticity 6.1.4, it will require only $1.8 \cdot 12 = 21$ kg., to stretch the steel wire as far as the 12 kg. stretch the copper wire, it will be seen that the limits of perfect elasticity for steel are twice as wide as they are for copper.

There are some substances whose elasticity, measured by the method of The_measure_of_elasticity 6.1.4 within very wide limits. India rubber is such a substance. When, in popular language, we speak of this substance as being very elastic, we have in mind the width of its elastic range rather than the numerical value of its elastid sonstant. In scientific discussion it it necessary to distinguish carefully between these two ideas. In this book a substance will be said to have a high elasticity only when it requires a large force to produce a small deformation.

6.1.6 Hooke's law

If we examine the stretches produced by the successive addition of kilogram masses in the experiment of section Elasticity 6.1.3, Fig 106, we shall find that these stretches are all equal, at least within the limits of observational error. Very carefully conducted experiments have shown that this law, namely that the successive application of equal forces produces a succession of equal stretches, holds very exactly for all sorts of elastic displacements, so long, and only so long, as the limits of perfect elasticity are not exceeded. This law is know as *Hooke's law*, after Englishman, Robert Hooke (1635-1703. Another way of stating this law is the following. *Within the limits of perfect elasticity elastic deformations* *of any sort, be they twists or bends or stretches, are directly proportional to the forces producing them.*

6.1.7 Molecular forces vs. molecular motions

The above experiments have shoen that when the molecules of a solid are pulled farther apart than their natural distances, they tend to come back to these distances. Precisely similar experiments on compression show that if they are pushed closer together than their natural distances, they tend to spring apart. Thus, if one attempts to compress a rubber ball, a steel ball, an ivory ball, or almost any sort of a solid body, as soon as the force is removed the body will return to its natural size unless the compression had been carried too far.

As a given temperature, then, the molecules of any solid tend to reamin a given distance apart and resist any attempt to increase to decrease this distance. The quesion whish at once suggests itself is, Why do not the attractive forces existing between the molecules pull them into the most intimate contact possible, so that no spaces whatever are left between them, and not compressing forces can presss them close together. The answer is found in the effects of heat on solid bodies. The molecules do in fact come closer together as soon as we lower the temperature, i.e. as soon as we decrease the velocity with which the molecules are oscillating back and forth within their little intermolecular spaces, and they push out to greater distances as soon as we raise the temperature. The size which a given solid body possesses at any given temperature is then the result of a balance between two opposing tendencies, one a tendency to come as close together as possible on account of the *attractions* of the molecules, and the other a tendency to expand indefinitely like gases, because of the *motions* of the molecules. If we diminish the motions by lowering the temperature, we destroy the balance and the forces pull the molecules closer together. If we increase the motions by raising the temperatue, we render them more effective than the attractive forces, and the body expands. So long, however, as the temperature remains constant any attmept to press the molecules closer together of push them farther apart is resisted, the one by the motions, the otehr bu the attractive forces.

6.1.8 Cohesion and adhesion

The preceeding experiments have brought out the fact that in the solid sondition, at least, molecules of the same kind exert attractive forces uon one another. That molecules of unlike substances also exert mutually attractive forces is equally true, as is proved by the fact that glue sticks to wood with tremendous tenacity, mortar to bricks, nickel plating to iron, etc.

The forces which bind *like* kinds of molecules together are commonly called *cohecive forces*; those which bind together molecules of *unlike* kind are called *adhesive forces.* Thus we say that mucilage sticks to wood because of *adhesion*, while wood itself holds together because of *cohesion.* Again, adhesion holds the chalk to the blackboard, while cohesion holds together the particles of the crayon.

Hardness
Brittleness
Ductility
Malleability

6.1.9 Properties of solids depending on cohesion

Many of the physical properties in which solid substances differ from one another depend on differences in the cohexive forces existing between the molecules. Thus we are accustomed to classify solids with relation to their hardness, brightness, ductility, maleability, tenacity, elacticity, etc. The last two of these terms have been sufficiently explained int he preceding paragraphs; but since confusiton sometimes arises from failure to understnad the first four, the tests for these properties are here given.

We test for relative *hardness* of two bodies by seeing which will *scratch* the other. Thus the diamond is the hardest of all substances, since is scratches all others and is scratched by none.

We test for relative *brittleness* of two substances by seeing which will *break* mose easily under a blow from a hammer. Thus glass and ice are very brittle substances; lead and copper are not.

We test the relative *ductility* of two bodies by seeing which can be *drawn into the thinner wire*. Platinum is the most ductile of all substances. It has been drawn into wires but 0.000762 mm. in diameter. Glass is also very ductile when sufficiently hot, as may be readily shown by heating it to softness in a Bunsen flame, when it may be drawn into threads which are o fine as to be almost invisible.

We test the relative *mallleability* of two substances by seeing which san be *hammered into the thinner sheet*. Gold, the most malleable of all substances, has been hammered into sheets 0.000085 mm in thickness.

6.1.10 Questions and problems

Add this section [2]

6.2 Molecular Forces In Liquids. Capillary Phenomena

6.2.1 Proof of the existence of molecular forces in liquids

The facility with which liquids change their shape might lead us to suspect that the molecules of such substances exert almost no forces upon one another, but a simple experiment will show that this is far from true

By means of sealing wax and string let a glass plate be suspended horizontally from one arm of a balance, as in Fig. 107. After equilibrium is obtained let a surface of water be placed just beneath the plate and the beam pushed down until contact is made. It will be found necessary to ass a considerable weight to

[2] Add in the footnote from page 107 of A First Course In Physics.

the opposite pan in order to pull the plate away from the water. Since a layer of water will be found to cling to the glass it is evident that the added force applied to the pan has been expended in pulling water molecules away from water molecules, not in pulling glass away from water. Similar experiments may be performed with all liquids. In the case of mercury the glass will not be found to be wet, showing that the cohesion of mercury is greater than the adhesion of glass and mercury.

Shape_assumed_by_a_free_liquid

6.2.2 Shape assumed by a free liquid

Since, then, every molecule of liquid is pulling on every other molecule, any body of liquid which is free to take its natural shape, i.e. which is acted on only by its won cohesive forces, must draw itself together until it has the smallest possible surface compatible with its volume; for, since every molecules in the surface is drawn toward the interior by the attraction of the molecules within, it is clear that molecules must continually move toward the center of the mass until the hole has reached the most compact form possible. Now the geometrical figure which has the smallest area for a given volume is a sphere. We conclude, therefore, that if we could relieve the body of liquid from the action of gravity and other outside forces, it would at once take the form of a perfect sphere. This conclusion may be easily verified by the following experiment.

let alcohol be added to water until a solution in which a drop of common lubricating oil will float at any depth. Then with a pipette insert a large globule of oil beneath the surface. The oil will be seen to float as a perfect sphere within the body of the liquid (Fig. 108). (Unless the drop is viewed from above, the vessel should have flat rather than cylindrical sides, otherwise the curved surfaces of the water will act like a lens and make the drop *appear* flattened.)

The reason that liquids are not more commonly observed to take the spherical form is that ordinarily the force of gravity is so large at to be more influential in determining their shape than tare the cohesive forces. As verification of this statement we have only to observe that as a body of liquid becomes smaller and smaller,–i.e. as the gravitational forces upon it become less and less,–it does indeed tend more and more to take the spherical form. Thus very small globules of mercury on a table will be found to be almost perfect spheres, and raindrops or minute *floating* particles of all liquids are quite accurately spherical.

Contractility_of_liquid_films

6.2.3 Contractility of liquid films

The tendency of liquids to assume the smallest possible surface furnishes a simple explanation of the contractility of liquid films.

Let a soap bubble five to seven centimeters in diameter be blown on the bowl of a pipe and then allowed to stand. It will at once begin to shrink in size in a few minutes will disappear within the bowl of the pipe. The liquid of the bubble is simply obeying the tendency to reduce its surface to a minimum, a tendency which is due to the mutual attractions which its molecules exert upon one another. A candle flame held opposite the opening in the stem of the pipe will be deflected by the current of air which the contradicting bubble is forming out through the stem.

Again, let a loop of fine thread be tied to the edge of a wire, as in Fig. 109. Let the ring be dipped into a soap solution so as to form a film across it, and then let a hot wire be thrust through the film inside the loop. The tendency of the film outside of the loop to contract will instantly spread out the thread into a perfect circle (Fig. 110). The reason that the thread takes the circular form is that since the film outside the loop is striving to assume the smallest possible surface, the are inside the loop must of course become as large as possible. The circle is the figure which has the smallest possible area for a given perimeter.

Let a soap film be formed across the mouth of a funnel, as in Fig. 111. The tendency of the film to contract will cause it to run quickly toward the small end of the funnel.

6.2.4 Ascension and depression of liquids in capillary tubes

It was shown in Chapter 3 that, in general, a liquid stands at the same level in any number of communicating vessels. The following experiments will show that this rule ceases to hold in the case of tubes of small diameter.

let a series of capillary tubes of diameter varying from 2mm. to 0.1 mm. be arranged as in Fig. 112.

When water is poured into the vessel it will be found to rise higher in the tubes than in the vessel, and it will be seen that the smaller the tube the greater the height to which it rises. If the water is replaced by mercury, however, the effects will be found to be just inverted. The mercury is depressed in all tubes, the depression being greater in proportion as the tube is smaller (Fig. 113). This depression is most easily observed with a U-tube like that shown in Fig. 114.

Experiments of this sort have established the following laws.

1. *Liquids rise in capillary tubes when they are capable of wetting them, but are depressed in tubes which they do not wet.*

2. *The elevation in the one case, and the depression in the other, are inversely proportional to the diameter of the tubes.*

It will be noticed, too, that when a liquid rides, its surface within the tube is concave upward, and when it is depressed its surface is convex upward.

6.2.5 Cause of curvature of a liquid surface in a capillary tube

All of the effects presented in the last paragraph can be explained by the consideration of cohesive and adhesive forces.

The second fact upon which the explanation will rest is one the truth of which was demonstrated by the spherical shape assumed by the very small globules of liquid (see section 6.2.2). It is that the force of gravity actin on a *very small* body of liquid is negligible in comparison with its own cohesive force.

These two points established, consider where water is in contact with the glass wall of the tube. Let the quantity of liquid considered be so minute that the force of gravity acting upon it may be disregarded. The force of *adhesion* of the wall will pull the liquid particles at o in the direction of oE. The force of *cohesion* of the liquid will pull these same particles in the direction of oF.

It was shown in Chapter 2 that if the lengths of the lines oE and oF are made proportional to the relative strengths of these two forces, the actual dircetion and magnitude of the resultant force will be represented by the direction and magnitude of the diagonal oB of the parallelogram of which oE and oF are two adjacent sides (Fig. 115).

If, then, the adhesive force oE greatly exceeds the cohesive force oF, the direction oR of the resultant force will lie to the left of the vertical om, in which case, since a liquid always sets itself so that its surface is at right angles to the resultant force, the liquid about o must set itself in the position shown in Fig. 116; i.e. it must rise up against the wall as water does against glass.

If the cohesive force oF (Fig. 117) is strong in comparison with the adhesive force oE, the resultant oR will fall to the right of the vertical, in which case the liquid must be depressed about o.

Whether, then, a liquid will rise against a solid wall or be depressed by it, will depend only on the relative strengths of the adhesive and cohesive forces that exist between he walls of the tube and the liquid itself. Since mercury does not wet glass we know that cohesion is here relatively strong, and we should expect, therefore, that the mercury would be depressed, as indeed we find it to be. The fact that water will wet glass indicates in this case adhesion is relatively strong, and hence we should expect water to rise against the walls of the containing vessel, as in fact it does.

It is clear that a liquid which is depressed near the edge of a vertical solid wall must assume within a tube a surface whish is convex upward, while a liquid which rises against a wall must within such a tube be *concave upward*.

6.2.6 Explanation of ascension and depression in capillary tubes

The fact that liquids assume curved surfaces within tubes makes it easy to see why a liquid which is concave must rise and one which is convex must fall. For, consider first a liquid which, because of the strength of the adhesion between it and the walls of the tube, assumes a *concave* surface within the tube (Fig. 118). It was shown in section 6.2.2 and 6.2.3 that the mutual attraction of the molecules of a liquid for one another always exhibits itself as a tendency to reduce the exposed surface of the liquid to a minimum. hence this concave surface aob (Fig 118) must tend to straighten out into the flat surface $ao'b$. But it no sooner thus begins to straighten out than adhesion again elevates it at the edges. it will be seen, therefore, that the liquid must continue to rise in the tube until the weight of the volume of liquid lifted, namely $amnb$ (Fig. 119), balances the tendency of the surface *aob* to move up. That the liquid will rise higher in a small tube that in a large one is to be expected, since the weight of the column of liquid to be supported in the small tube is less.

Precisely the same method of reasoning applied to the convex mercury surface *aob* (Fig. 120) shoes why the mercury must fall in a capillary tube until the upward pressure at o, due to the depth h of mercury (Fig. 121), balances the tendency of the surface *aob* to flatten out.

6.2.7 Capillary phenomena in everyday life

Capillary phenomena play a very important part in the process of nature and of everyday life. Thus the rise of oil in wicks of lamps, the complete wetting of a towel when one end of it is allowed to stand in a basin of water, the rapid absorption of liquid by limp of sugar when one corner of it only is immersed, the taking up of ink by blotting paper, are all illustrations of precisely the same phenomena which we observe in the capillary tubes of Fig. 112.

6.2.8 Floating of small objects on water

Let a needle be laid very carefully on the surface of a dish of water. In spite of the fact that it is nearly eight times as dense as water it will be found to float. If the needle has been perviously magnetized, it may be made to move about in any direction over the surface in obedience to the pull of the magnet.

To discover the cause of this apparently impossible phenomenon, examine closely the surface of the water in the immediate neighborhood of the needle. It will be found to be depressed in the manner shown in Fig. 122. This furnishes at once the explanation. So long as the needle is so small that its own weight is no greater than the upward force exerted upon it by the tendency of the depressed (and therefore concave) liquid surface to straighten out into a flat surface, the needle could not sink in the liquid, no matter how great its density. If the water had wetted the needle, i.e. if it had risen about the needle instead of being depressed, the tendency of the liquid surface to flatten out would have pulled it down into the liquid instead of forcing it upward. Any body which a liquid is depressed will therefore float on the surface of the liquid if its mass is not too great. Even if the body, when perfectly clean, causes the liquid to ride about it, an imperceptible film of oil on its surface will cause it to depress the liquid, and hence to float.

The above experiment explains the familiar phenomenon of insects walking and running on the surface of water (Fig. 123) in apparent contradiction to the law of Archimedes, in accordance with which they should sink until they displace their own weight of the liquid.

6.2.9 Questions and problems

Add this section

6.3 Solution and Crystallization

6.3.1 Solution and molecular force

let a speck of permanganate of potash, about as big as a pin head, be dropped into a quart flask full of water. The water will at once begin to be colored about the particle, and in a short time the particles itself will have completely disappeared. After a little shaking the whole body of water will have acquired a rich red tint.

This process of the solution of solids in liquids, so familiar to us from the use of salt and sugar in liquid foods, furnishes a good illustration of the differences in the attraction which the molecules of the same liquid exert on the molecules of different solids, or which the molecules of the same solid exert on those of different liquids. At ordinary temperatures water dissolves three times as much common table salt as does alcohol, and it dissolves gum arabic quite readily, whereas alcohol scarcely dissolves it at all. On the other hand, resin, shellac, etc., are readily soluble in alcohol, but quite insoluble in water. Benzene and gasoline are used for removing grease spots from clothing, because most forms of grease, although insoluble water, are readily soluble in these liquids. Beeswax, which is not appreciable dissolved by water, alcohol, or benzene, is quite readily dissolved in turpentine.

From these facts it is clear that *adhesive forces* have much to do with the process of solution. On the other hand, the *motions of the molecules* must also be intimately concerned with this process, for we have seen that the facts of the evaporation of ice and of other solids prove that even where there are no adhesive forces pulling the molecules of a solid from one another, the motions alone cause some of them to escape form the surface and pass off into the space above. This tendency to pass off must be present as well when the space is filled with liquid as when it is empty.

6.3.2 Saturated solutions

The last conclusion is confirmed when we find that in many respects solution is analogous to evaporation. Just as at a given temperature only a certain amount of liquid will evaporate into a closed space, so also there is a definite limit to the amount of a solid which will dissolve at any temperature in a given body of liquid. This is proved by the familiar fact that after a certain amount of sugar has been added to a cup of coffee, further addition simply deposits so much more sugar in the bottom of the cup. At ordinary temperatures the maximum amount of common salt which can be made to dissolve in 100ml of water is about 36g.

Now just as a vapor which has reached its highest possible density is called a saturated vapor, so a solution which contains as large an amount of a solid as it is capable of taking up is called a *saturated solution*.

6.3.3 Saturation and temperature

ation_and_temperature

In the last chapter it was found that a liquid or a solid evaporates more readily at a high temperature tan at a low one,–a fact which is readily explained by the theory that an increase in temperature means an increase in the average velocity of the molecules. It is to be expected from the same theory that increase in temperature will increase the ease with which a solid substance goes into solution in a liquid. For, as suggested above, the increased motions can be no less effective in causing molecules to leave the solid and pass off into the space above when that space is filled with a liquid than when it is empty. In the former case the adhesive force and the motion of the molecules *together* effect the disintegration of the solid, while in the latter the motions are only agents at work.

As a matter of fact, experiment shows that it is true, in general, that solids are dissolved much more readily in hot liquids than in cold ones. It is for no other reason than this that hot water is so much more effective than cold for cleaning purposes. The amount of potassium nitrate (saltpeter) that will dissolve in 100ml of water a given temperatures is listed in Figure 6.1. Saltpeter_Solution

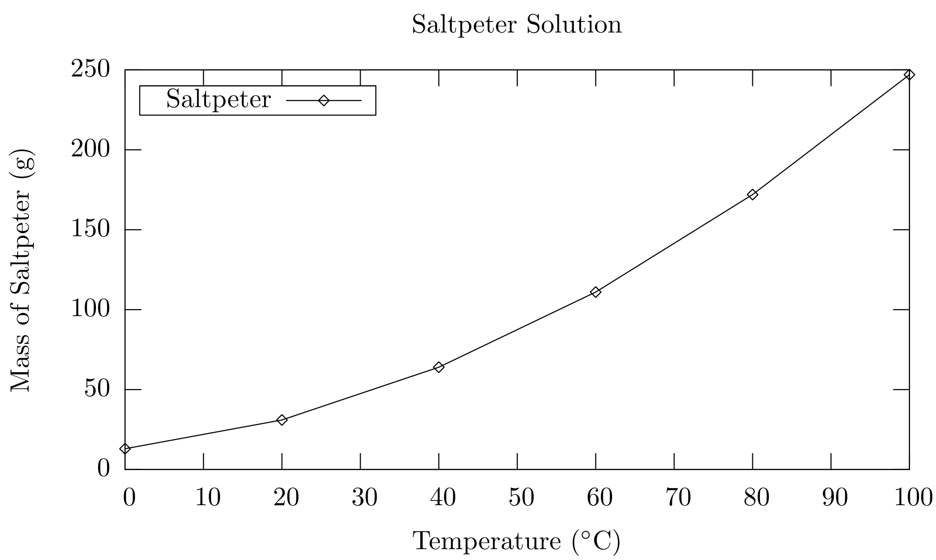

Figure 6.1: Saltpeter Solution

Saltpeter_Solution

6.3.4 Effect of evaporating a solution

When a solution evaporate it is, in general, the liquid only which passes off into a vaporous condition, practically all of the dissolved substance remaining

Crystallization

behind. This is proven by the fact that the rain which falls at sea-level is fresh water and not salt water, and by the fact that impurities are removed from water by distillation.

6.3.5 Effect of evaporating a saturated solution or of lower its temperature

If a saturated solution is evaporating, it must soon become more than saturated, for the same amount of dissolved substance remains, while the volume of the solution continually diminishes. The result is exactly what would be expected from the analogy between solution and evaporation. It will be remembered that when the volume of a saturated vapor was diminished a part of the vapor condensed. So when the saturated solution evaporates the dissolved substance gradually seperates out inthe *solid* condition. This is illustrated by the fact that the evaporation of salt sea spray leaves the face and clothing covered with salt. It is the process of evaporating sea water that is frequently used to obtain table salt.

Again, just as there is a second way of causing a saturated vapor to condense, namely by lowering its temperature, so lowering the temperature of a saturated solution will also caure the molecules of the dissolved substance to pass out of solution and to collect in the solid form.

6.3.6 Crystallization

If the separation of solid from solution is made to take place slowly and quietly, by either of the above methods, the beautiful and striking phenomena of *crystallization* may be observed. The molecules of the separating solid group themselves in regular geometric forms. These forms vary greatly with the nature of the dissolved substance, thus indicating differences in the nature of he cohesive forces which act to bring the molecules together.

Thus if a saturated solution of common salt is filtered and then set aside, after twenty-four hours groups of crystals will be found floating on the surface. If one of these is carefully removed and examined with a magnifying glass, the crystals will be found to be perfect little cubes.

Again, if a thread be hung in a beaker or large test tube containing a saturated solution of alum, in a few days the thread will be covered with octahedral crystals (Fig. 124) about the size of a pea.

If copper sulphate be treated in the same way, large blue crystals of the form shown in Fig. 125 will collect on the thread

If a hot saturated solution of saltpeter (potassium nitrate) be placed in a beaker and closely watched as it cools, it will be found possible to actually see the process and growth of crystals of the form in Fig. 126.

Wherever the crystals are in contact with the sides of the vessel the free formation is interfered with and the resulting forms are very irregular.

Most minerals are found on microscopic study to have a crystalline structure, though in nature they have usually been formed under conditions which render it impossible for the crystals to be large and regular.

Diamond is carbon crystalized under conditions which once existed in nature by which man has been able to reproduce in the laboratory only upon a very diminutive scale

6.4 Absorption of Gases by Liquids

6.4.1 Absorption of gases by liquids

Let a large test tube be filled with ammonia gas by heating aqua ammonia and causing the evolved gas to displace mercury in the tube, as in Fig. 127. let a piece of charcoal an 2.5cm long and nearly as wide as the tube be heated to redness and then plunged beneath the mercury. When it is cool let it be slipped underneath the mouth of the tst tube and allowed to rise into the gas. The mercury will be seen to rise in the tube, as in Fig. 128, thus showing that the gas is being absorbed by the charcoal. If the gas is unmixed with air, the mercury will rise to the bery top of the tube, thus whowing that all the ammonia has been absorbed by the charcoal.

This property of absorbing gases is possessed to a notable degree by porous substances, such as charcoal, meerschaum, gypsum, silk, etc. It is not improbable that all solids hold, closely adhering to their surfaces, thin layers of the gases with which they are in contact, and that the prominence of the phenomena of absorption in porous substances is due to the great extent of surface possessed by such substances.

That the same substance exerts widely different gases is shown by the fact that charcoal will absorb 90 times its own volume of ammonia gas, 35 times its volume of carbon dioxide, and but 1.7 times its volume of hydrogen. The usefulness of charcoal as a deodorizer is due to its enormous ability to absorb certain kinds of gases.

6.4.2 Absorption of gases in liquids

Let a beaker containing cold water be slowly heated. Small bubbles of air will be seen to collect in great numbers upon the walls and rise through the liquid to the surface. That they are indeed bubbles or air and not oof steam is proved first by the fact that they appear when the temperature is far below boiling, and second by the fact that they do not condense as they rise into the high and cooler laywers of water.

The experiment shows two thing,–first, that water of dinarily contains considerable quantities of air dissolved in it, and second, that the amount of air which water can hold decreases as the temperature rises. The first point is also probed by the existence of fish life, for fishes obtain the oxygen which they need to support life, not immediately from the water but from the air which is dissolved in it.

The amount of gas which will be absorbed by water varies greatly with the nature of the gas. At 0°C and 76cm barometer height 1ml of water will absorb 1050 ml of ammonia, 1.8ml of carbon dioxide, and but 0.4 ml of oxygen. Ammonia itself is a gas under ordinary conditions. The commercial aqua ammonia is simply ammonia gas dissolved in water.

The following experiment illustrates the absorption of ammonia by water.

Let the flask F (Fig. 129) and tube b be filled with ammonia by passing a current of the gas in at a and out through b. Then let a be corked up and b thrust into G, a flask nearly filled with water which has become colored slightly red by the addition of litmus and a drop or two of acid. As the ammonia is absorbed the water will slowly rise in b, and as soon as it reaches F it will rush up very rapidly until the upper flask is nearly full. At the same time the color will change from red to blue because of the action of the ammonia upon the litmus.

Experiment shows that *inevery case of absorption of a gas by a liwuid or solid, the quantity of gas absorbed decreases with an increase in temperature.*–a result which was to have been expected from the kinetic theory, since increasing the molecular velocity must of course increase the difficulty which the adhesive forces have in retaining the gaseous molecules.

It will be noticed that the effect of temperature upon solution (section 6.3.3) Saturation_and_temperature is quite the opposite of its effect upon absorption.

6.4.3 Effect of pressure upon absorption

Soda water is ordinary water whish has been made to absorb large quantities of carbond dioxide gas. This impregnation is accomplished by gringing the water into contact with the gas under high pressure. As soon as the pressure is relieved the gas passes rapidly out of solution. This is the sause of the characteristic effervescence of soda water. These facts show clearly that the amount of carbon dioxide which can be absorbed by water is greater for high pressures than for low. As a matter of fact, careful experiments have shown that the amount of any gas absorbed is directly proportional to the pressure, so that if carbon dioxide under a pressure of 10 atmospheres is brought into contact with water, 10 times as much of the gas is absorbed as if it had been under a pressure of 1 atmosphere.

6.4.4 Questions and problems

Add this section

Chapter 7

Thermometry-Expansion Coefficents

7.1 Thermomentry

7.1.1 Meaning of temperature

When a body feels hot to the touch we are accustomed to say that it has *high temperature*, and when it feels cold that it has a *low temperature*. Thus the word "temperature" is used to denote the condition of hotness or coldness of the body whose state is being described.

7.1.2 Measurement of temperature

So far as we know, up to the time of Galileo no one had ever used any special instrument for the measurement of temperature. People knew how hot or how cold it was from their feelings only. But under some conditions this temperature sense is a very unreliable guide. For example, if the hand has been in hot water, the same tepid water will feel warm; a room may feel hot to one who has been running, while it will feel cool to one who has been sitting still.

Difficulties of thie sort led to the introduction in modern times of mechanical devices for measuring temperature, called *thermometers*. These instruments depend for the operation upon the fact that practically all bodies expand as they grow hot.

7.1.3 Galileo's thermometer

It was in 1592 that Galileo, at the University of Padua in Italy constructed the first thermometer. He was familiar with the facts of expansion of solids, liquids, and gases; and, since gases expand more than solids or liquids, he chose a gas as his expanding substance. His device was that shown in Fig. 130.

The relative hotness of two bodies was compared by observing which one of the two, when placed in contact with the air bulb, caused the liquid to descend farther in the stem S. As a matter of fact, berometric changes as well as temperature changes cause changes in the height of the iquid in the stem of such an instrument, but Galileo does not seem to have been aware of this fact.

it was not until about 1700 that mercury thermometers were invented. On account of their exterme convenience these have now replaced all others for pratical purposes.

7.1.4 The construction of a Centigrade mercury thermometer

a_Centigrade_mercury_thermometer

The meaning of a degree of temperature chance is best understood from a description of the method of making and graduating a mercury thermometer.

A bulb is blown at one end of a piece of thick-walled glass tubing of small uniform bore. Bulb and tube are then filled with mercury, at a temperature slightly above the highest temperature for which the thermometer is to be used, and thte tube is sealed off in a hot flame. As the mercury cools it contracts and falls away from the top of the tube, leaving a vacuum above it.

The bulb is next surrounded with melting now or ice, as in Fig. 131, and the point at which the mercury stands in the tube is marked 0°. Then the bulb and tube are placed in the steam rising fromboiling water, as in Fig, 132, and the new position of mercury is marked 100°. The space between these two marks on the stem is then divided into 100 equal parts, and divisions of the same length are extended above the 100° mark and below the 0° mark.

One degree of change in temperature, measured on such a thermometer, means, then, such a temperature change as will cause the mercury in the stem to move over one of these divisions; i.e. it is such a temperatuer change as will cause mercury contained in a glass bulb to expand $\frac{1}{100}$ of the amount which it expands in passing from the tempeture of melting ice to that of boiling water. A thermometer in which the scale is divided in this way is called a Centigrade thermometer.

Thermometers graduated on the Centigrade scale are used almost exclusively in scientific work, and also for ordinary purposes in most countries which have adopted the metric system. This scale was first devised in 1742 by Celsius, of Upsala, Sweden. For this reason it is sometimes called the Celsius instead of Centigrade scale.

7.1.5 Fahrenheit thermometers

The common household thermometer in England and the United States differs from the Centigrade only in the manner of its graduation. In its construction the temperature of melting ice is marked 32° instead of 0°, and that of boiling water 212° instead of 100°. The intervening stem is then divided into 180 parts. The zero of this scale is the temperature obtained by mixing equal weights of sal ammoniac (ammonium chloride) and snow. In 1714, when Fahrenheit, of

Danzig, Germany, devised this scale, he chose this zero because he thought it represented the lowest possible temperature, i.e. the entire absence of heat.

7.1.6 Comparison of Centigrade and Fahrenheit thermometer

From the methods of graduation of the Fahrenheit and Centigrade thermometers it will be seen that 100° on the Centigrade scale denotes the same difference of temperature as 180° on the Fahrenheit scale (Fig. 133). Hence one Fahrenheit degree is equal to five ninths of a centigrade degree, and ont centigrade degree is equal to nine fifths of a Fahrenheit degree. *Hence to reduce from the fahrenheit to the Dentigrade scale, first find how many fahrenheit degrees the given temperature is above or below the freexing temperature, and then multiply by five ninths.*

To reduce from Centigrade to Fahrenheit, first multiply by nine fifths in order to find how many Fahrenheit degrees the given temperature is above or below the freezing temperature. Knowing how far it is from the freezing point, it will be very easy to find how far it is from 0° F., which is 32° below the freezing point.

7.1.7 The range of the mercury thermometer

Since Mercury freezes at −39° C., temperature lower than this are very often measured by means of *alcohol* thermometers, for the freezing point of alcohol is −130.5° C. Similarly, since the boliing point of mercury is 360° C., mercury thermometers cannot be used for measuring very high temperature. For both very high and very low tmeperatures, in fact all temperatures, a *gas* thermometer is the standard instrument.

7.1.8 The standard hydrogen thermometer

_hydrogen_thermometer

The modern gas thermometer, however (Fig. 134), is widely different from that devised by Galileo (Fig 130). It is not usually the increase in the volume of a gas kept under constant pressure which is taken as the measure of temperature change, but rather the increase in pressure which the molecules of a confined gas exery against the walls of a vessel whose volume is kept constant. The essential features of the method of calibration and use of the standard hydrogen thermometer at the International Bureau of Weights and Measures at Paris are as follows.

The bulb B (Fig. 134) is first filled with hydrogen and the space above the mercury in the tube a made as nearly a perfect vaccum as possible. B is then surrounded with melting ice (as in Fig. 131) and the tube a raised or lowered until the mercury in the arm b stands exactly opposite the fixed mark c on the tube. Now, since the space above D is a vacuum, the pressure exerted by the hydrogen in B against the mercury surface at c just supports the mercury column ED The point D is marked on a strip of metal behind the tube a. The bulb B is then placed in a steam bath like that shown in Fig. 132. The increased pressure of the gas in B at once begins to force the mercury down at c and up at D. But by raising the arm a the mercury in b is forced back again to c, the increased pressure of the gas on the surface of the mercury at c begin balanced by the increased height of the mercury column supported, which is now EF instead of ED. When the gas in B is thoroughly heated to the temperature of the steam, the arm a is very carefully adjusted so that the mercury in b stands very exactly at c, its original level. A second mark is then placed on the metal strip exactly opposite the new level of the mercury, i.e. at F. D is then marked 0° C., and F is marked 100° C. The vertical distance between these marks is divided into 100 exactly equal parts. Divisions of exactly the same length are carried above the 100° mark and below the 0° mark. One degree of change in temperature is then defined as any change in temperature which will cause the pressure of the gas in B to change by the amount represented by the distance between any two of these divisions.

In other words, *one degree of change in temperature is such a temperature change as will cause the pressure exerted by a confined gas to change* $\frac{1}{100}$ *as much as it changes in passing between the temperature of melting ice and boiling water.*

To find any unknown temperature, e.g. the temperature of liquid air, it is only necessary to immerse the bulb B in the liquid air, adjust the arm a until the mercury in b stands on the scale behind a. If this reading is 180 divisions below the point D, the temperature of liquid air is $-180°$ C.

7.1.9 The pressure coefficient of expansion of a gas

The ratio between the increase in the pressure in B per degree rise in temperature and the total pressure which the gas in B exerts at 0° C. is called the pressure coefficient of expansion of the gas. Thus, for exampe, the pressure coefficient is $\frac{1}{100}$ of $\frac{DF}{ED}$. Now $\frac{DF}{ED}$ is found to be exactly $\frac{100}{273}$, or 0.367. Hence the pressure coefficient of hydrogen is $\frac{1}{273}$, or 0.00367. To state the definition in the form of an equation, let P_t be the pressure at $t°$ C. and P_0 at 0° C.; then the increast in pressure has been $P_t - P_0$, the increase per degree has been $\frac{P_t - P_0}{t}$, and since the prssure coefficient c is this increase divided by P_0, we have

$$c = \frac{P_t - P_0}{P_0 t} \tag{7.1}$$

7.1.10 The law of Charles

In 1787 a Frenchman, Charles discovered that *the pressure coefficients of all gases are the same*; i.e. they are all $\frac{1}{273}$. This is now called the *the law of charles*. it follows from this law that any gas thermometer may be constructed as a standard thermometer, since all gas thermometers must agree with one another in their readings.

7.1.11 Comparison of gas and marcury thermometers

Since an international committee has chosen the hydrogen thermometer described in section The_standard_hydrogen_thermometer 7.1.8as the standard temperature measurement, it is important to know whether mercury thermometers, graduated in the manner described in section The_construction_of_a_Centigrade_mercury_thermometer 7.1.4, agree with gas thermometers at temperatures other than 0° and 100°, where, of course, they must agree, since these temperatures are in each sace the starting points of the graduation. A careful comparison has shown that slthough they do not agree exactly, yet fortunately the disagreements at ordinary temperatures are small, not amounting to more that 0.2° anywhere between 0° and 100°. At 300° C., however, the difference amounts to 4°.

hence for all ordinary purposes mercury thermometers are sufficiently accurate, and no special standardization of them is necessary. But in all scientific work, if mercury thermometers are used at all, the must first be compared with a gas thermometer and a table of corrections obtained. The errors of an alcohol thermometer are considerably larger than those of a mercury thermometer.

7.1.12 Absolute temperature

It is clear from a description of the method of graduation and using the standard gas thermometer (section The_standard_hydrogen_thermometer 7.1.8) that we take as the measure of temperature the pressure which the body of air confined in B exerts, i.e. the heigt of the column of mercury above E. A certain increase in this height, namely an increase equal to $\frac{1}{100}$ of DF, means, by defiition, 1° rise in temperature, and a decrease in this height equal to $\frac{1}{100}$ of DF, means 1° fall in temperature. Now since this distance, $\frac{1}{100}$ of DF, is $\frac{1}{273}$ of ED, i.e. $\frac{1}{273}$ of the pressure which the gas exerts at 0°, it follows that if the temperature could be cooled 273° below 0° C., the level in a would be exactly the same as the level in b; i.e. the gas in the bulb would exert *no* pressure. But, from the standpoint of kinetic theory, a gas must always exert pressure so long as its molecules are in motion. Hence the temperaure at which its molecules cease to exert pressure, viz. $-273°$ C., is the temperature at which it molecules cease to move. This temperature is called the *absolute zero*, because no lower temperature can possible exist, since it is impossible to conceive that the gas in B can exert a pressure less than zero, i.e. that that level in a can never fall below the level of c. A scale of temperature is now often used in which this point, namely $-273°$ C., is taken as the zero. This is called the *absolute scale*, and temperatures expressed in terms of this scale are called *absolute temperatures*. Thus if A. is used to denote absolute temperatures and C. to denote Centigrade temperatures, we have $0° = 273°\text{A}, 100°\text{C.} = 373\text{A.}, 15°\text{C.} = 288°$ A., etc. It is customary to indicate temperatures on the Centigrade scale by t and temperatures on the absolute scale by T. We have therefore

$$T = t + 273 \tag{7.2}$$

7.1.13 Low temperatures

The absolute zero of temperature can, of course, never be attained, but in recent years rapid strides have been made toward it. Twenty-five years ago the lowest temperature which had ever been measured was $-110°$ C., the temperature attained by Faraday in 1845 by causing a mixture of ether and solid carbon dioxide to evaporate in a vasuum. But in 1880 air was first liquified, and found, by means of a gas thermometer, to have atemperature of $-180°$ C. When liquid air evaporates into a space which is kept exhausted by means of an air pump, its temperature falls to about $-220°$ C. Recently hydrogen has been liquefied and found to have a temperature of $-243°$ C. All of these temperatures have been measured by means of hydrogen thermometers. By allowing liquid hydrogen to evaporate into a space kept exhausted by an air pump, Dewar in 1900 attained a temperature as low as $-260°$ C., only $13°$ C. above the absolute sero.

7.1.14 Maximum and minimum thermometers

In all weather bureaus the lowest temperature reached during the night, and the highest temperaure reached during the day, are automatically recorded by a special device called a maximum and minimum thermometer. The construction of oe form of this instrument is shown in Fig. 135.

The bulb A and the stem down to the point G are filled with alcohol, from G to B the stem is filled with mercury, whild the liquid above B is again alcohol. The bulb D contains only aocohol and its vapor. The two indices d and C move with slight friction in the stem. As the temperature reached. As the temperature rises the alcohol in A expands and the mercury pushes up the index on the left and leaves it opposte the mark corresponding to the highest temperature reached. In order to obtain the right amount of friction a small steel spring is atached to the indices, as in K. After each observation the observer pulls the index back to contact with the mercuty by means of a small magnet.

7.1.15 Questions and problems

Add this section.

7.2 Expansion Coefficient of gases Gas

7.2.1 Volume coefficient of expansion of a gas

oefficient_of_expansion_of_a_gas

When we measure the pressure coefficient of expansion of a gas we keep the volume constant (Fig. 134) and observe the rate at which the pressure increases with a rise in temperature. If, however, we arrange the experiment so that the gas can expand as the temperature rises, the pressure remaining constant, we obtain what is called *the volume coefficient of expansion*. It is defined as the *ratio between the increase in volume per degree and the total volume of the gas at zero.*

It was a Frenchman, Gay-Lussac, who in 1802 first made measurements upon this quantity and found that *all gases have the same volume coefficient of expansion*, this coefficient being the same as the pressure coefficient, anmely $\frac{1}{273}$.

7.2.2 Volume proportional to absolute temperature if pressure is constant

It will be seen from the discussion of the gas thermometer (Fig. 134) that if the volume of a gas is kept constant, *the pressure is proportional to the absolute temperature*; for by definition absolute temperature is measured by the difference in the heights of the two mercury columns in a and b; but this same difference is also a measure of the pressure in B. To state thie relation algebraically, let P_1 and P_2 be the pressures at the two absolute temperatures T_1 and T_2 respectively, then

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \tag{7.3}$$

P1/P2=T1/T2

Now since, by section Volume_coefficient_of_expansion_of_a_gas 7.2.1, the rate of change of volume at constant pressure is the same as the rate of change of pressure at constant volume, we have also, when the pressure is constant and the volume changes from V_1 to V_2,

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}; \tag{7.4}$$

V1/V2=T1/T2

i.e. *the volume of a gas at constant pressure is directly proportional to the absolute temperature.*

Or, again, since volume is always inversely proportional to density, we at once, from equation V1/V2=T1/T2 7.2.2, that *at constant pressure the density of a gas is inversely proportional to its absolute temperature*; i.e.

$$\frac{D_1}{D_2} = \frac{T_2}{T_1}; \tag{7.5}$$

D1/D2=T2/T1

7.2.3 Questions and problems

Add this section

Chapter 8

Work and Mechanical Energy

8.1 Definition and Measurement of Work

8.1.1 Definition of work

WHenever a force *moves* a body on which it acts, it is said to do work upon that body; and the amount of the work accomplished is measured by the product of the force acting and the distance through which it moves the body. Thus if 1 g. of mass is lifted 1 cm. in a vertical direction, 1 g. of force has acted, and the distance through which it has moved the body is 1 cm. We say, therefore, that the lifting force has accomplished 1 *gram centimeter* of work. If the gram of force had lifted the body upon which it acted through 2cm., the work donw would have been 2 b. cm. If a force of 3 g. had been acted and the body had been lifted through t cm., the work done would have been 9 g. cm., etc. Or, in general, if W represents work accomplished, F the vlaue of the acting force, and D the distance through which its point of application moves, then the definition of work is given by the equation

$$W = \vec{F}d \tag{8.1}$$

W=Fd

In the scientific sense, no work is ever done unless the force succeeds in *producting motion* in the body on which it acts. A pillar supporting a building does no work; a man tugging at a stone, but failing to move it, does no work. In the popular sense we sometimes say that we are doing work when we are simply holding a weight, or doing anything else which results in fatigue; but in physics the work "work" is used to describe, not the effort put forth, but the *effect accomplished*, as represented in equation 8.1.1.

Erg
Joule

8.1.2 Units of work

Corresponding to the two metric units of force, the gram of force and the dyne (see section The_dyne ??, there are two metric units of work, the gram centimeter and the dyne centimeter, the latter of which is usually called the *erg*.

The *gram centimeter* is the amount of work done by 1 gram of force when it moves the point upon which it acts 1 cm.

The *erg* is the amount of work done by 1 dyne of force when it moves the point upon which it acts 1 cm.

The erg is called an absolute unit of work for the reason that it involves in its definition the absolute unit of force, namely the dyne. To raise 1 l. of water from the floot to a table 1 m. high would require $1000 \cdot 9.8 \cdot 100 = 98,000$ ergs. It will be seen, therefore, that the erg is an exceedingly small unit. For this reason the *joule*, in honor of the great English physicist, James Perscott Joule (1818-1889). The work done in lifting a liter of water one meter is therefore 0.98 of a joule.

Corresponding to the English unit of dorce, the pound, we have the english unit of work, the foot pound, which is the amount of work done by a "pound of force" when it moves the point on which it acts through a distance of one foot.

8.1.3 Questions and problems

Add this section

8.2 Work and Pulleys

8.2.1 The single fixed pulley

let the force of the eatrh's attraction upon a mass W be overcome by pulling upon a spring balance S, in the manner shown in Fig. 146, until W moves slowly upward. If W is 100 g., the spring balance will also be found to register a force of 100 g.

Experiment therefore shows that in the use of the single fixed pulley the acting force F which is producing the motion is equal tothe resisting force W which is opposing the motion.

Again, since the length of the string is always constant, the distance s through which the point A, at which F is applied, must move, is always equal to the distance s' through which the weight W is lifted. Hence, if we consider the work put into the system at A, viz. Fs, and the work accomplished by the system at W, viz/ Ws', we find obviously, since $W = F$ and $s = s'$, that

$$Fs = Ws' \tag{8.2}$$

; ie.e in the case of the single fixed pulley, *the work done by the acting force F is equal to the work done against the rsisting force W*; or the work *put into* the machine at A is equal to the work *accomplished by* the machine at W.

8.2.2 The single movable pulley

Let now the force of the earth's attraction upon the mass W be overcome by a single movable pulley, as shown in Fig. 147. Since the weight of W (W representing in this case the weight of both the pulley and the suspended mass) is now supported half vy the strand C and half by the strand B, the force F which must act at A to hold the weight in place, or to move it slowly upward if there is no friction, should be only one half of W. A reading of the balance will show that this is indeed the case.

Experiment thus shows that in the case of the single movable pulley the acting force F is just one half as great as the resisting force W.

But when again we consider the *work* which the force F must do to lift the weight W a distance s', we see that A must move upward 2 in. in order to raise W 1 in. For when W moves up 1 in. both of the strands B and C must be shortened one inch. As before, therefore, since $W = 2F$, and $s' = \frac{1}{2}s$,

$$Fs = Ws'$$

; i.e. in the case of the single movable pulley, as in the case of the fixed pulley, *the work put into the machine at F is equal to the work accomplished by the machine at W.*

8.2.3 Combinations of pulleys

Let a weight W be lifted by means of such a system of pulleys as is shown in Fig. 148, either (1) or (2). Here, since W is supported by 6 strands of cord, it is clear that the force which must be applied at A in order to hold W in place, or to make it move slowly upward if there is no friction, should be $\frac{1}{6}$ of W.

The experiment will show this to be the cause if the effects of friction, which are often very considerable, are eliminated by taking the mean of the forces which must be applied at F to cause it to move first slowly upward and then slowly downward. The law of combination of movable pulleys may then be stated thus: *If n represent the number of strands between the weight is divided,*

$$F = \frac{W}{n} \tag{8.3}$$

But when again we consider the work which the force F must do inorder to lift the weight W through a distance s', we see that, in order that the weight W may be moved up through 1 in., each of the strands must be shortened 1 in., and hance the point A must move through n in.; i.e. $s' = \frac{s}{n}$. hence, ignoring friction, in the case also we have

$$Fs = Ws'$$

; i.e. although the acting force F is only $\frac{1}{n}$ of the resisting force W, *the work put into the machine at F is equal to the work accomplished by the machine at W.*

Lever" Arm
Moment

8.2.4 Mechanical advantage

The above experiments show that it is sometimes possible, by applying a small force F, to overcome a much larger resisting force W. *The number of times by which the resisting force W exceds the applied force F is called the mechanical advantage of the machine.* Thus the mechanical advantage of the single fixed pulley is 1, that of the single movable pulley is 2, that of the systems of pulleys shown in Fig. 148 is 6, etc.

if the acting force is applied at W instead of at F, the mechanical advantage of the systems of pulleys of Fig. 148 is $\frac{1}{6}$; for it requires an application of 6 lb. at W to lift 1 lb. at F. But it will be observed that the resisting force at F now moves six times as fast and six times as far as the acting force at W. We can thus either sacrifice suppd to gain force, or sacrifice force to gain speed; but in every case, whatever we gain in the one we lose in the other. thus in the hydraulic elevator shown in Fig. 40, page 48, the cage moves only as fast as the piston; but in that shown in Fig. 41 it moves four times as fast. Hence the force applied to the piston in the latter case must be four times as great as in the former if the same load is to be lifted. This means that the diameter of the latter cylinder must be twice as great.

8.2.5 Questions and problems

Add this section.

8.3 Work and the Lever

8.3.1 The law of the lever

The lever is a rigid rod free to turn about some point P called the fulcrum (Fig. 149).

Let a meter stick be first balanced as in the figure, and then let a mass, of say 3 kg, be hung by a thread from a point 15 cm. from the fulcrum. Then let a point be found on the other side of the fulcrum at shich as weight of 100 g will just support the 300 g This point will be found to be 45 cm from the fulcrum. It will be seen at once that the product of $300 \cdot 15$ is equal to the product of $100 \cdot 45$.

Next let the point be found at shich 150 g. just balance the 300 g. This will be found to be 30 cm. from the fulcrum. Again the products $300 \cdot 15 = 150 \cdot 30$.

No matter where the weights are placed, or what weights are used on either side of the fulcrum, the product of the acting force F by the distance l from teh fulcrum (Fig. 150) will be found to be equal to the product of the resisting force W by its distance l' from teh fulcrum. Now the distances l and l' are called the *lever arms* of the forces F and E, and the product of a force by its lever arm is called the *moment* of that force. The above experiments on the lever may then be generalized in the following law. *the moment of acting force is equal to the* *moment of the resisting force.* Algebraically stated, it is

$$F \cdot l = W \cdot l' \tag{8.4}$$

Fl=Wl' .

It will be seen that the *mechanical advantage* of the lever, namely $\frac{W}{F}$, is equal to

8.3.2 Addition of moments

Let 200 g., for example, be placed 30 cm. from P (Fig. 151), and on the other side 100 g, 20 cm from P, and let the point be found at which another 100 g mass must be placed in order to produce equilibrium. This point will be found to be 40 cm from P, and it will be seen that $200 \cdot 30 = 100 \cdot 20 + 100 \cdot 40$; i.e. that th emoment on the left is equal to the sum of the moments on the right.

Next, let the lever be arranged as in Fig. 152, and let 300 g. be hung at A, 30 cm from the fulcrum; 100 g at B, 15 cm from the fulcrum; and 50 g hung over the pully h, the thread being attached at a distance 40 cm from the fulcrum. Then let the point be found at which a weight of 200 g will produce equiligrium. This point will be 32.5 cm from the fulcrum.

It will be seen that

$$300 \cdot 20 + 50 \cdot 40 = 100\ 15 + 200 \cdot 32.5$$

; i.e. *the sum of al lthe moments which are tending tomake the beam rotate in one direction is equal to the sum of all the moments tending to make it rotate in the opposite direction.* This is the general statement of the law of the lever.

8.3.3 Work expected uon and accomplished by the lever

We have just seen that when the lever is in wquilibrium–that is, when it is *at rest* or is *moving uniformlly*–the relation between the acting force F and the resisting force W is expressed in the equation of moments, viz. $Fl - Wl'$. Let us now suppose, precisely as in the case of the pulleys, that the force F raises the weight W through a small distance s'. To accomplish this, the point A to which F is attached must move through a distance s (Fig. 153). From the similarity of the triangles APn and BPn it will be seen that $\frac{l}{l'}$ is equal to $\frac{s}{s'}$. hence equation Fl=Wl' 8.3.1, which represents the law of the lever, and which may be written $\frac{F}{W} = \frac{s'}{s}$, or

$$Fs = Ws'$$

Now Fs represents the work donw by the acting force F, and Ws the work donw against the resisting force W. Hence the law of moments, which has just been found by experiment to be the law of the lever, is equivalent to the statement that *whatever work is accomplished by the use of the lever, the work expended upon the lever by the acting force F is equal to the work accomplished by the lever against the resisting force W.*

8.3.4 The three classes of levers

Is it customary to divide lever into three classes, as follows.

1. In levers of the first class the fulcrum P is between the acting force F and thre resisting force W (Fig. 154. The mechanical advantage of levers of this class is greater or less than unity, according as the lever arm l of the acting forst is greater or less than the lever arm l' of the resisting force.

2. In levers of the second class the resisting force W is between the acting force F and the fulcrum P (Fig. 155). here the lever arm of the acting force, i.e. the distance from F to P, is necessarily greater than the lever arm of the resisting force, i.e. the distance from W to P Hence the mechanical advantage is always greater than one.

3. In levers of the third class the acting force is between the resisting force and the fulcrum (Fig. 156). The mechanical advantage is then obviously less than one, i.e. in this type of lever force is always sacrificed for the sake of speed.

8.3.5 Questions and problems

Add this section.

8.4 The Principle of work

8.4.1 Statement of the principle of work

The study of pulleys led us to the comclusion that in all cases where such machines are used the work done by tha acting force is equal to the work done against the resisting force, provided always that the motions are uniform, and that friction may be neglected. The study of levers led to precisely the same result. In Chapter II the study of the hydraulic press showed that the same law aplied in this case also, for it was shown that the force on the small piston times the distance through which it moved was equal to the force on the large pison times the distance through which it moved. Similar experiments upon all sorts of machines have shown that in all cases where friction may be neglected the following is an absolutely general law: *In all mechanical devices of whatever sort the work expended upon the machine is equal to the work accomplished by it.*

This important generalization is called "the principle of work," and was first enunciated by Sir Isaac newton in 1687, in a scholium to the third law of motion. It has proved to the one of the most fruitful principles ever put forward in the history of physics. By its application it is easy to deduce the relation between the force applied and the force overcome in any sort of machine, provided only that friction is negligible, and that the motions take place slowly. it is only necessary to produce, or imagine, a displacement at one end of the machine, and then to measure or calculate the corresponding displacement at the other end. The ratio of the second displacement to the first is the ratio of the force acting to the force overcome.

8.4.2 The wheel and axle

Let us apply the workk principle to discover the law of the wheel and axle (Fig. 158). When the large wheel has made one revolution the point A moves down a distance equal to the circumference of this wheel. During this time the weight W is lifted a distance equal to the circurmference of the exle. Hence the equation $Fs = Ws'$ becomes $F \cdot 2\pi R = W \cdot 2\pi r$, where R and r are the radii of the wheel and axle respectively. This equation may be written in the form

$$\frac{W}{F} = \frac{R}{r} \tag{8.5}$$

i.e. the weight lifted on the axel is as many times the force applied to the wheel as the radius of thw wheel is times the radius of the axle. Otherwise stated, *the mechanical advantage of the wheel and axle is equal to the radius of the wheel divided by the radius of the axle.*

The *capstan* (Fig 159) is a special case of the wheel and axle, the length of the leer arm taking the place of the radius of the wheel, and the radius of the barrel corresponding to the radius of the axle.

8.4.3 The work principle applied to the inclined plane

The work againse gravity in lifing a weight W (Fig. 160) from the bottom to the top of a plane is evidently equal to W times the gheight h of the plane. But the work done by the acting force F, while the carriage of weight W is being pulled from the bottom to the top of the plane, is equal to F times the length l of the plane. Hence the principle of work gives

$$Fl = Wh, \text{ or } \frac{W}{F} = \frac{l}{h} \tag{8.6}$$

i.e. *the mechanical advantage of the inclined plane, or the ratio of the weight lifted to the force acting parallel to the plane, is the ratio fot he length of the plane to the height of the plane.* This is precisely the conclusion at which we arrived in another way in Chapter II (p. 19).

8.4.4 The screw

The screw (Fig. 161) is a combination of the inclined plane and the lever. Its law is easily obtained from the principle of work. When the force which acts on the eid of the lever has moved this point through one complete revolution, the weight W, which rests on top of the screw, has eivdently been lifted through a vertical distance equal to the distance between two adjoining threads. This distance d is called the *pitch* of the screw. Hence, if we represent by l th length of the lever, the work principle gives

$$F \cdot 2\pi l = Wd; \tag{8.7}$$

i.e. *the mechanical advantage of the screw, or ratio of weight lifted to the force applied, is equal to the ratio of the circumference of the circle moved over by the end of the lever, to the distance between the threads of the screw.* In actual practice the friction in such an arrangement is always very great, so that the mechanical advantage is considerably less than its full theoretical value. The common jackscrew just described, and used chiefly for raising buildings, the letter press (Fig. 162), and the vice (Fig. 163) are all familiar forms of the screw.

8.4.5 A train of gear wheels

A form of machine capable of very high mechanical advantage is the train gear wheels shown in Fig. 164. Let the student show from the principle of work, namely $Fs = W's$, that the mechanical advantage, i.e. $\frac{W}{F}$, of such a device is

$$\frac{\text{circum. of } a \cdot \frac{\text{no. of cogs in } d}{\text{no. of cogs in } e} \cdot \frac{\text{no. of cogs in } f}{\text{no. of cogs in } d}}{\text{circum. of } e} \tag{8.8}$$

8.4.6 The worm gear

Another device of high mechanical advantage is the worm gear (Fig. 165). Show that if l is the length of the crank arm C, n the number of theeth in the cog wheel W, and r the radius of the axle, the mechanical advantage is given by

$$\frac{2\frac{\pi}{n}}{2\pi r} = n\frac{l}{r} \tag{8.9}$$

This device is used most frequently when the primary object is to decrease speed rather than to multiply force. It will be seen that the crank handle must make n turns while the cog wheel makes one.

8.4.7 The differential pulley

In the differential pulley (Fig. 166) an endless chain passes firse over the fixed pulley A, then down over the movable pulley C, then upagain over the fixed pulley B, which is rigidly attached to A, but differs slightly from it in diameter. On the circumference of all the pulleys are projections which fit between the links, and thus keep the chains from slipping. When the chain is pulled down at F, as in Fig. 166 (2), until the upper rigid system of pulleys has made one complete revolution, the chain between the upper and lwer pulleys has been shortened by the difference between the circumferences of the pulleys A and B, for the chain has been pulled up a distance equal to thecircumference of the larger pulley and let down a distance equal to the circumference of the smaller

Horse power
Watt

pulley. Hence the load W has been lifted by half the difference between the circumferences of A and B. The mechanical advantage is therefore equal to the circumference of A divided by one half the difference between the circumferences of A and B.

8.4.8 Questions and problems

Add this section.

8.5 Work and Energy

8.5.1 Definition of power

When a given load has been raised a given distance a given amount of work has been done, whether the time consumed in doing it is small or great. Time is therefore not a factor which enters into the determination of work; but it is often as important to know the *rate* at which work is done as to know the *amount* of work accomplished. *The rate of doing work is called power.* Thus, if P represent popwer, W the work down, and t the time required to do it,

$$P = \frac{W}{t} \tag{8.10}$$

8.5.2 Horse power

James Watt (1736-1819), the inventor of the steam engine, considered that an average horse could do 33,000 ft·lb of work per minute, or 550 ft·lb per second. The metric equivalent is 76.05 $\frac{\text{kg}\cdot\text{m}}{\text{s}}$. This number is probably considerably too high, but it has been taken ever since, in English-speaking countries, as the unit of power, and named *horse power* (H.P.). The power of steam engines has usually been rated in horse power. The horse power of an ordinary railroad locomotive is from 500 to 1000. Stationary engines and steamboat engines of the largest size often run from 5000 to 20,000 H.P. The power of an average horse is about 3/4 H.P., and that of an ordinary man about 1/7 H.P.

8.5.3 The kilowatt

In the metric system the erg has been has been taken as the absolute unit of work. The ocrresponding unit of power is an erg per second. This is, however, so small that it is customary to take as the pratical unit 10,000,000 ergs per second, i.e. one joule per second (see 3.5, page 38). This units is called the *watt*, in honor of James Watt. the power of dynamos and electric motors is almost always expressed in kilowatts, a kilowatt representing 1000 watts, and in modern practice even steam engines are being increasingly rated in kilowatts rather than in horse power. A horse power is equivalent to 746 watts; it may therefore in general be considered to be 3/4 of a kilowatt.

Energy

8.5.4 Definition of energy

The *energy* of a body is defined as its *capacity for doing work*. In general, inanimate bodies pssess energy only because of work which has been done upon them as some previous time. Thus, suppose a kilogram mass is lifted from the first position in Fig. 172 through a height of one meter, and placed upon the hook H at the end of a cord which passes over a frictionless pulley p and is attached at the otther end to a second kilogram mass B. The operation of lifting A from position 1 to position 2 has required an expenditure upon it of 1 kg·m of work. But in position 2, A is itself possessed of a certain capacity for doing work which it idid not have before. For if it is now started downward by the application of the slightest concievable force, it will, of its own accord, return to position 1, and will in so doing raise the likigram weight B through a height of 1 m. In other words, it will do upon B exactly the same amount of work which was originally done upon it.

8.5.5 Potential and kinetic energy

A body may have a capacity for doing work not only because it has been given an elevated position, but also because it has in some way axquired velocity: e.g. a heavy fly wheel will keep machinery running for some time after the power has been shut off; a bullet shot upward will lift itself a great distance against gravity because of the velocity which has been imparted to it. Similarly, any body which is in motion is able to rise against gravity, or to set other bodies in motion by colliding against them, or to overcome resistances of any concievable sort. Hence, in order to distinguish between the energy whish a body may have because of an *advantageous position*, and the energy which it may have because of its *motion*, the two terms "potential" and "kinetic" energy are used. Potential energy includes the energy of lifted weights, of colied or stretched springs, of bent bows, etc.; in a word, it is *energy of position*, while kinetic energy is *energy of motion*.

8.5.6 Transformations of potential and kinetic energy

The swinging of a pendulum, and the oscillation of a weight attached to a spiing, illustrate well the way in which energy which has once been put into a body may be transformed back and forth between the potential and kinetic varieties. When the pendulum bob is at rest at the bottom of its arc it possesses no energy of either type, since, on the one hand, it is as low as it can be, and on the other, it has not velocity. When we pull it up the arc to position A (Fig. 173), we do an amount of work upon it which is equal in gram centimeters to its weight in grams times the distance AD in centimeters; i.e. we store up in it this amount of potential energy. As now the bob falls to C this potential energy is completely transformed into kinetic. That this kinetic energy at C is exactly equal to the potential energy at A is proved by the fact that if friction is completely eliminated, the bob rises to a point B such that BE is equal to AD.

We see, therefore, that at the ends of its weing the energy of the pendulum is all potential, while in the middle of the swing its energy is all kinetic. In intermediate positions the energy is part potential and part kinetic, but the sum of the two is equal to the original potential energy.[1]

8.5.7 General statement of the law of frictionless machines

In our development of the law of machines, which led us to the conclusion that the work of the acting force is always equal to the work of the resisting force, we were careful to make two important assumptions,–first, that friction was negligible, and second that the motions were all either uniform or so slow that not appreciable velocities were imparted. In other words, we assumed that the work of the acting force was expended simply in ligting weights of compressing springs, i.e. in storing up potemtial energy. If now we drop the second assumption, a very simple experiment will show that our conclusion must be somewhat modified. Suppose, for instance, that instead of lifting a 500p weight slowly by means of a balance, we jerk it up suddenly. We shall now find that the initial pull indicated by the malance, instead of being 500 g will be consicerably more,–perhaps as much as several thousand grams if the pull is sufficiently sudden. This is obviously because the acting force is not overcomeing not merely the 500g which represents the resistance of gravity, but also the inertia of the body, since velocity is being imparted to it. now work done in imparting belocity to a body, i.e. in overcoming its inertia, always appears as *kinetic* energy, while work done in overcoming gravity appears as the potential energy of ligting weight. hence, whether the motions produced by machines are slow or fast, if friction is negligible, the law for all devices for transforming work may be stated thus: *The work of the acting force is equal to the sum of the potential and kinetic energies stored up in the mass acted upon.* In machines which work against gravity the body usually starts from rest and is left at rest, so that the kinetic energy resulting from the whole operation is zero. hence in such cases the work done is the weight lifted times the height through which it is lifted, whether the motion is slow or fast. The kinetic energy imparted to the body in starting is all given up by it in stopping.

8.5.8 The measure of potential and kinetic energy

The measure of the potential energy of any lifted body, such as a lifted pile driver, is equal to the work which has been spent in the lifted body. Thus if h is the height in meters, and m is the body/s mass, g is the force of gravity, then the potential energy E_p of the lifted mass is

$$E_p = mgh \, \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2} = mgh\mathrm{J} \tag{8.11}$$

[1] It is recommended that a laboratory exercise on the laws of the pendulum precede this discussion. See, for example, Experiment 17, authors' manual.

Since this energy is all transformed into kinetic energy when the mass falls the distance h, the product mF_gh also represents the number of joules of kinetic energy which the moving weight has when it strikes the pile.

If we wish to express this kinetic energy in terms of the velocity of which the weight strikes the pile, instead of the height from which it has falled, we have only to substitute for h its value in terms of g and the velocity acquired (see equation 3.5,, 38), namely $h = \frac{v^2}{2g}$. This gives the kinetic energy E_k in the form

$$E_k = \frac{1}{2}mv^2\mathrm{J} \tag{8.12}$$

Since it makes no difference how a body has acquired its velocity, this represents the general formula for the kinetic energy *in joules* of any moving body, in terms of its mass and its velocity.

Thusthe kinetic energy of a 1 g (0.001 kg) bullet moving with a velocity of $100\frac{\mathrm{m}}{\mathrm{s}}$ is

$$E_k = \frac{1}{2} \cdot 0.001 \cdot 100^2 = 5J$$

.

We know, therefore, that the posder pushing on the bullet as it moved through the rifile barrel did 51.02 kilogram meters of work upon the bullet in giving it a velocity of 100 $\frac{\mathrm{M}}{\mathrm{s}}$

8.5.9 Questions and problems

Add this section

Chapter 9

Work and Heat Energy

9.1 Friction

9.1.1 Friction always results in wasted work

All of the experiments mentioned in the last chapter were so arranged that *friction* could be neglected or eliminated. So long as this condition was fulfilled it was found that the result of universal experience could be stated in the law, *The work done by the acting force is equal to the sum of the kinetic energy and potential energies stored up.* In other words, if there were no friction, no work would ever be wasted. We should be able to obtain from every machine exactly as much work as we put into it, no more and no less.

But whereever friction is present this law is found to be inexact, for the work of the acting force is then always somewhat greater than the sum of the kinetic and potential energies stored up. If, for example, a block is pulled over the horizontal surface of a table, at th end of the motion no velocity has been imparted to the block, and hence no kinetic energy has been stored up. Further, the block has not been lifted not put into a condition of elastic strain, and hence no potential energy has been communicated to it. We cahhot in any way obtain from the bolck more work agter the motion htan we could have obtained before it was moved. It is clear, therefore, that al of the work which was done in moving the block against the friction of the table was *wasted work*. Experience shows that, in genera, where work is doen against friction it can never be regained. Before considering what becomes of this wated work we shall consider some of the factors on which friction depends, and some of the laws which are found by experiment to hold in cases in which friction occurs.

9.1.2 Laws of sliding friction

The unavoidable irregularities in all surfaces make accurate experiments upon friction impossible. The following laws are therefore to be regarded as rough apporximations. They may easily be verified in a general way by pulling a block

Friction!Coefficient of

ofer a smoor board with the aid of a spring balance, or by means of a pulley and weights arranged as in Figure 174.

1. The friction between two solid surfaces is greater at starting than after the motion has begun. This is doubtless because the inequalities in the upper surface sink into those in the lower and more completely at rest than in motion.

2. After the motion has started the friction between solid surfaces is independent of the speed of the motion.

3. Friction between two surfaces is proportional to pressure; i.e. doubling the weight W of the block and load together (Fig. 174) makes it necessary to double the force F required to maintain uniform motion.

4. Friction is independent of extent of surface so long as the total force pressing the two surfaces together is constant; i.e. it requires the same force to keep a brick sliding on its edge as on its side. This result follows necessarily from 3.

9.1.3 Coefficient of friction

From 3 and 4 of the preceeding section it follows that if F (Fig. 174) is the force necessary to maintain uniform motion in the weight W, then the ratio F/W depends only on the nature of the two surfaces in contact. It is called the *coefficiten of friction* for the given materials. Thus if F is 300 g and W is 800 g, then the coefficient of friction between the block is $\frac{300}{800} = 0.375$. The coefficient of iron upon iron is about 0.2, of oak on oak about 0.4.

9.1.4 Rolling friction

The chief cause of sliding friction is the interlocking of minuts projections (shown greatly magnified at a, b, c, and d in Fig. 175). When a round solid *rolls* over a smooth surface the frictional resistance is generally much less than when it slides; e.g. the coefficient of friction of cast-oron wheels rolling on iron rails may be as low as 0.002, i.e. $\frac{1}{100}$ of the sliding friction of iron on iron. This means that a pull of 1 lb will keep a 500-lb car in motion. Sliding friction is not, however, entirely dispensed with in ordinary wheels, for although the rim of the wheel rolls on the track, the axle slides sontinususly at the same point c (Fig. 176) upon the surface of the journal.

The great advantage of the bal bearing (Fig. 177) is that the sliding friction in the hub is almost completely replaced by rolling friction.

9.1.5 Fluid friction

When a solid moves through a fluid, as when a bullet moves through the air or a ship through the water, the resistance encountered is not at all independent of velocity, as in the case of colid friction, but increases for slow speeds nearly as the square of the velocity, and for high scpeeds at a rate considerably greater. This explains why it is so expensive to run a fast train; for the resistance of the ari, which is a smal lpart of the total resistance so long as the train is moving slowlh, becomes the predominant factor at high speeds. The resistance offered to steamboats running at high speeds is usually considered to increase as the cube of the velocity. Thus the *Cedric*, of the White Star Line, having a speed of 17 knots, has a horse power of 14,000, and a total weight when loaded of about 38,000 tons, while the *Kaiser Wilhelm II*, of the Hamburg-American Line, having a speed of 25 knots, has an engine of 40,000 horse power, although the total weight when loaded is only 26, 000 tons.

9.1.6 Internal friction

When a lead bullet strikes against a target the layers of lead slip over one another in the process of flattening, and thus the total kinetic energy of the bullet is wasted in internal friction within the lead. This waste of energy because of internal friction takes plase, to some extent, whenever a body is distorted, but in elastic bodies the waste is much less rapid than in inelastic ones. Thus when a rubber hall is dropped upon a stone sidewalk, the kinetic energy of the ball just before impact is largely transformed into potential energy of strain, and this is again transformed into kinetid energy in the rebound. But since the ball will never rebound quite to its original height, we know that there is in this case also a certain amount of energy wasted in internal friction within the ball. In general, the greater the amount of *permanent deformation* the greater the waste in internal friction.

9.1.7 Questions and problems

Add this section.

9.2 Efficiency

9.2.1 Definition of efficiency

Since it is only in an ideal machine that there is not friction, in all actual machines the work done by the acting force always exceeds, by the amount of the work doen againse friction, the amount of potential and kinetic energy stroed up. We have seen that the latter is wasted work in the sense that it can never be regained. Sice the energy stored up represents work which can be regained, it is termed *useful work*. In most machines an effort is made to have the useful work as large a fraction of the total work expended as possible. *The ratio of the useful work to the total work doen by the acting force is called the "efficiency" of the machine.* Thus

$$\text{Efficiency} = \frac{\text{Useful work accomplished}}{\text{Total work expended}} \tag{9.1}$$

Tus, if in the system of pulleys shown in Fig. 148 it is necessary to add a weight of 50g at F in order to pull up slowly an added weight of 240 g at W, the work done by the 50 g while F is moving over 1 cm will be $50 \cdot 1$ g·cm. The useful work accomplished in the same time is $240 \cdot \frac{1}{6}$ g·cm. Hence the efficiency is equal to $\frac{240 \cdot \frac{1}{6}}{50 \cdot 1} = \frac{4}{5} = 80\%$.

9.2.2 Efficiencies of some simple machines

in simple levers the friction is generaly so small as to be negligible; hance the efficiency of such a machine is approximately 100%. When inclined planes are used as machines the friction is also small, so that the efficienty generally lies between 90% and 100%. The efficienty of the commercialblock and tackle (Fig. 148), with several movable pulleys, is usually considerably less, varying between 40% and 60%. In the jackscrew there is necesarily a very large amount of friction, so that although the mechanical advantage is enormous, the efficiency is often as low as 25%. The differential pully of Fig. 166 has also a very high mechanical advantage with a very small efficiency. Gear wheels such as those shown in Fig. 164, or chain gears such as those used in bicycles, are machines of comparitively high efficiency, often utilizing between 90% and 100% of the energy expended upon them.

9.2.3 Efficiency of overshot water wheels

The overshot water wheel (Fig. 178) utilizes the potential energy of the water at S; for the wheel is turned by the weight of the water in the buckets. The work expended on the wheel per second, in ft·lb or g·cm, is the product of the weight of the water which passes over it per second by the distance through which it falls. The efficiency is the work which the wheel can accomplish in a second, divided by this quantity. Such wheels are very common in mountainous regions, where it is easy to obtain considerable fall, but where the streams carry a small *volume* of water. The efficiency is high, being often between 80% and 90%. The loss is due not only to the friction in the bearings and gears (see C), but also to the fact that some water is spilled from the buckets, or passes over without entering them at all. This may still be regarded as a frictional loss, since the energy disappears in internal friction when the water strikes the ground.

9.2.4 Efficiency of undershot water wheels

The old-style undershot wheel (Fig. 179), so common in flat countries where there is little fall but abundance of water, utilizes only the kinetic energy of the water running through the race from A. It seldom transforms into useful work more than 25% or 30% of the potntial energy of the water above the dam. There are, however, certain modern forms of undershot wheel which are extremely efficient. For examply, the *Pelton wheel* (Fig. 180), developed since 1880, and now very commonly used for small-power purposes in cities supplied with waterworks, sometimes has an efficienty as high at 83%. The water is delivered from a nozel o against cup-shaped buckets arranged as in the figure.

9.2.5 Efficiency of water turbines

The turbine wheel was invented in France in 1833, and is now used more than any other form of water wheel. It stands comlpetely under water in a case at the bottomg of a *turbine pit*, rotating in a horizontal plane. Fig. 181 shows one of the several methods of installing such a wheel. AB is the turbine pit and C the outer case into which the water enters from the pit. Fig. 182 shows the outer case with contained turbine; Fig. 183 is the inner case in which are the fixed guides G, which direct the water at the most advantageous angle against the blades of the wheel inside; Fig. 184 is the wheel itself; and Fig. 185 is a section of wheel and inner case, showing how the water enters through the guides and impinges upon the blades W. The spent water simply falls down the blades into the tailrace (Fig 181). The amount of water which passes through the turbine can be controlled by means of the rod P (Fig. 182, which can be turned so as to increast or decrease the sze of the openings between the guides G (Fig. 183). The energy expended upon the turbine per second is the product of the mass of water which passes through it by the height of the turbine pit. Efficiencies as high at 90% have been attained with such wheels. The most powerful turbine is in existence is at Shawenegan Falls, Quebec, Canada. The pit is 135 ft deep, the wheel 10 ft in diameter, and the horse power developed 10,500.

9.2.6 Questions and Problems

Add this section.

9.3 Mechanical Equivalent of Heat

9.3.1 What becomes of wasted work?

In all of the devices for transforming work which we have considered we have found that on account of frictional resistances a certain per cent of the work expended upon the machine is wasted. The question which as once suggests itself is, "What becomes of this wasted work?" The following familiar facts suggest an answer. When two sitcks are vigorously rubbed toegther they become hot; augurs and drills often become too hot to hold; matches are ignited by friction; if a strip of lead be struck a few sharp blows with a hammerj, it is apprecialby warmed. Now since we learned in Chapter V that, according to modern notions, increasing the temperature of a body means simply increasing the average velocity of its molecules, and therefore their average kinetic energy, the above facts point strongly to the conclusion that in each case the mechanical energy expended has been simply transformed into the energy of molecular motion. This view was first brought into prominence in 1798 by Benjamin Thompson, Count Rumford, an American by birth. It was first carefully tested

Calorie

by the English physicisy, James Prescott Joule (1818-1889), in a series of epoch-making experiments extending from 1842 to 1870. In order to understand these experiments we must first learn how heat quantities are measured.

9.3.2 Unit of heat,–the calorie

A unit of heat is defined as *the amount of heat which is required to raise the temperature of 1 g of water through* $1^\circ C$. This units is called *the calorie*. Thus, for example, when a hundred grams of water has its temperature raised four degrees, we say that four hundred calories of heat have entered the water. Similarly, when a hundred grams of water has its temperature lowered ten degrees, we say that a thousand calories have passed out of the water. If, then, we wish to measure, for instance, the amount of heat developed in a lead bullet when it strikes against a target, we ahve only to let the spent bullet fall into a known mass of water and to measure the number of degrees through which the temperature of the wtaer rises. The product of the number of grams of water by its rise in temperature is then, bu deginition, the number of calories of heat which have passed into the water.

it will be noticed that in the above definition we make no assumption whatever as to have *heat* is. Previous to the nineteenth century physicists generally held it to be an invisible, weightless fluid, the passage of which into or out of a body caused it to grow hot or cold. This view accounts well enough for the heating which a body experiences when it is held in contact witha flame of other hot body, but it has difficulty in emplaining the heating produced by rubbing of pounding. Rumford's view accounts easily for the heating of cold bodies by contact with hot ones; for we have only to think of the hotter and therefore more energetic molecules of hot body as communicating their energy tot he molecules of the colder body in much the same way in which a rapidly moving billiard ball transfers part of its kinetic energy to a more slowly moving ball against which it strikes.

9.3.3 Joule's experiment on the heat developed by friction

Joule argued that if the heat produced by friction, etc., is indeed merely mechanical energy which has been transferred to the molecules of the heated body, then the same number of calories must always be produced by the disappearance of a given amount of mechanical energy. And this must be true no matter whether th work is expended in overcoming the friction of wood on wood, of iron on iron, in percussion, in compression, or in any other conceivable way. To see whether or not this were so he caused mechanical energy to disappear in as many ways as possible and measured in every case the amount of heat developed.

In his first experiment he caused paddele wheels to rotate in a vessel of water by means of falling weights W (Fig. 186). The amounf of work done by gravity upon the weights in causing them to descend through and distance d was equal to their weight W times this distance. if the weights descended slowly and uniformly, this work was all expended in overcoming the resistance of the water to the motion of the paddle wheels though it; i.e. it was wasted in eddy currents in the water. Joule measured the rise in the temperature of the water and found that the mean of his three best trials gave 427 gram· meters as the amount of work required to develope enough heat to raise a gram of water one degree. He then repeated the experiment, substituting mercury for water, and obtained 425 gram·meters as the work necessary to produce a calorie of heat. The difference between these numbers is less than was to have been expected from the unaviodable errors in the observations. He the devised an arrangement in which the heat was developed by the friction of iron on iron, and again obtained 425.

9.3.4 Heat produced by collision

A Frenchman by the name of Hirn was the first to make a carefyl determination of the relation between the heat developed by *collision* and the kinetic energy which disappears. He allowed a steel cylinder to ball through a known height and crush a lead ball by its impact uon it. The amount of heat developed in the lead was measured by observing the rise in temperature of a small amount of water into which the lead was quickly plunged. As the mean of a large number of trials he, also, found that 425 gram·meters of energy disappeared for each calorie of heat which appeared.

9.3.5 Heat produced by the compression of gas

Another way in which Joule measured the relation between heat and work was by compressing a gas and comparing the amount of work done in the compression with the amount done in the compression of the amount of heat developed.

Every bicyclist is aware of the fact that when he inflates his tires the pump grows hot. This is due to partly to the friction of the piston against the walls, but chiefly to the fact that the downward motion of the piston is transferred to the molecules which come in contact with it, so that the velocity of these molecules is increased. The principle is precisely the same as that involved in the velocity communicated to a ball by a bat. If the bat is held rigidly fixed and a ball thrown against it, the ball rebounds with a certain velocity; but if the ball is moving rapidly forward to meet the ball, the latter rebounds with a much greater velocity. So the molecules which in their natural motions collide with an advancing piston, rebound with greater velocity than they would if they had impinged upon a fixed wall. THis increase is the molecular velocity of a gas on compression is so great that when a mass of gas at 0°C is compressed to one half its volume the temperature rises to 87°C.

The effect may be strikingly illustrated by the fire syringe (Fig. 187). Let a few drops of carbon bisulphide be placed on a small bit of cotton, dropped to the bottom the tube A, and then removed; then let the piston B be inserted and very suddenly depressed. Sufficient heat will be developed to ignite the vapor and a flash will result. (If the flash does not result from the first stroke, withdraw the piston completely, then reinsert, and compress again.)

To measure the heat compression Joule surrounded a small compression pump with water, took 300 strokes on the pump, and measured the rise in temperature of the water. As the result of these measurements he obtained 444 gram·meters as the "mechanical equivalent" of the calorie. The experiment, however, could not be performed with great exactness.

9.3.6 Cooling by expansion

Joule also obtained the reation between heat and work from experiments on the cooling produced by expansion. This process is exactly the conberse of heating by compression. If a compressed gas is allowed to expand and force out a piston, or merely to expand against atmospheric pressure, it is always found to be cooled by the process. This is because the kinetic energy of the molecules is transferred to the piston, so that the former revound from the latter with less velocity than they had before the blow. The refrigerators used on shipboard are good illustrations of this principle. Air is compressed by an engine to perhaps one fourth its natural volume. The heat generated by compression is then removed by causing the air to circulate about pipes kept cool by the flow of cold water through them. This compreseed air is then allowed to expand into the refrigerator chamber, the temperature of which is thus lowered many degrees.

Joule's determination of the mechanical equivalent of heat from the amount of work down by an expanding gas and the amount of heat lost in expansion gave 437 gram·meters. This experiment also was one for which no great amount of exactness could be claimed.

9.3.7 Significance of Joule's experiments

Joule made three other determinations of the realtion between heat and work by methods involving electrical measurements. He published as the man of all his dterminations 426.4 gram·meters as the mechanical equivalent of the calorie. But the value of his experiments does not lie primarily in the accuracy of te final results, but rather in the proof which they for the first time furnished that whenever a given amount of work is wasted, no matter in what particular way this waste may occur, there is always an appearance of the same definite invariable amount of heat.

The most accurate determination of mechanical equivalent of heat was made by Rowland (1848-1901) in 1880 at Johns Hopkins University. He obtained 427 g·m, or $419 \cdot 10^7$ ergs.

9.3.8 The conversion of energy

We are now in a position to state the law of all machines in its most general form, i.e. in such a way as to include even the cases where friction is present.

It is: *The work down by the acting force is equal to the sum of the kinetic and potential energies stored up plus the mechanical equivalent of heat developed*

In other words, *whenever energy is expended on a maching or device of any kind, an exactly equal amount of energy always appear either as useful work or as heat*. The useful work may be represented in the potential energy of the lifted mass, as when water is pumped up to a ressrvior; or in the kinetic energy of a moving mass, as when a stone is thrown from a sling; of in the potential energies of molecules whose positions with reference to one another have been changed, as when a spring has been bent; or in the molecular potential energy of chemically seperated atoms, as when an electric current seperates a compund substance. The *wasted* work always appears in the form of increased molecular motion, i.e. in the form of heat. This important generalization has received the name of the *Principle of Conservation of Energy*. It may be stated thus: *Energy may be transformed, but it can never be created or destroyed.*

9.3.9 Perpetual motion

In all ages there have been people who have spent their lives trying to invent a machine out of which work could be continually obtained, without the expenditure of an equivalent amount of work upon it. Such theoretical devices are called perpetual-motion machines. even to this day the United States patent office annually recives scores of applications for patents on such devices. The possibility of the existence of such a device is absolutely denied by the statement of the principle of the conservation of energy. For only in case there is no heat developed, i.e. in case there are no frictional losses, can work taken out be equal to the work put in, and in no case can it be greater. Since, in fact, there are always some frictional losses, the principle of the conservation of energy asserts that it is impossible to make a machine which will keep itself running forever, even though it does no useful work; for no matter how much kinetic or potential energy is imparted to the machine ito begin with, there much always be a continual drain upon this energy to overcome frictionalresistances; so that, as soon as the wasted work has become equal to the initial energy, the machines must stop.

The first man to make a formal and complete statement of the principle of the conservation of enerfy was the German physician, Robert Mayer, whose work was published in 1842. Twenty years later, partly through the theoretical writings of Helmholtz and Clausius in Germany, and of Kelvin and Rankine in England, but more especially through the experimental work of Joule, the principle had gained universal recognition and had taken the place which it now holds as the corner stone of all physical science.

9.3.10 Examples of transformation of energy

When a bullet is fired vertically from a rifile the energy which projects it upward first exists in the form of gunpowder. Gunpoweder is a mixture from 70%

to 80% potassium nitrate (niter) and 10% to 15% each of sulphur and charcoal. These elements combine in the explosion so as to form a mixture of the gases nitrogen and carbon dioxide. These gases occupy an atmospheric pressure about 1500 times the volume of the gunpowder from which they are formed. At the instant of formation they therefore possess the potential energy of highly compressed elastic bodies. In the process of expanding, this energy is transformed into kinetic energy of the rising bullet. In the ascent this kinetic energy is transformed into the potential energy of a lifted mass, and in the descent this potential energy is again transformed into kinetic energy. Finally, as the bullet strikes the earth this kinetic energy is all changed into heat, i.e. into molecular kinetic energy.

Ths transformation of energy which take place in any power plant, such as that in Niagra, are as follows. The energy firse exists as the potential energy of the water at the top of the falls. This is transformed in the turbine pits into the kinetic energy of the rotating wheels. Thses turbines drive dynamos in which there is a transformation into the energy of electric currents. These currents are carried by wires to Burffalo and other cities, where they run street cars and other forms of motors. The principle of conservation of energy asserts that the work which gravity did upon the water in causing it to descend from the top to the bottom of the turbine pits is exactly equal to the work done by all the motors, plus the heat developed in all the wires and bearings, and in the eddy currents in the water.

9.3.11 Energy derived from the sun

let us next consider where the water at the top of the falls obtained its potential energy. Whater is being continually evaporated at the surface of the ocean by the sun's heat. This heat imparts sufficient kinetic energy to the molecules to enable them to break away from teh attractions of their fellows and to rise above the surface in the form of vapor. The lifted vapor is carried by winds over the continents and precipitated in the form of rain or snow. Thus the potential energy of the water above the falls at Niagra is simply transformed heat energy of the sun. If, in this way, we analyze any available source of energy at man's disposal, we find in practically every case that it is directly traceable to the wun's heat as its source. Thus the energy contained in coal is simply the energy of separation of the oxygen and carbon which were seperated in the processes of growth. This seperation was effected by the sun's rays.

We can form some conception of the enormous amount of energy which the sun radiates in the form of heat by reflecting that os this heat the earth recieves not more than $\frac{1}{2000{,}000{,}000}$ part. Of the amount received by the earth not more than $\frac{1}{1000}$ part is stroed up in anomal and vegetable life and lifted water. This is practically all of the energy which is available on the eatrh for man's use.

9.3.12 Questions and problems

9.4 Specific Heat

9.4.1 Two ways of heating a body

In the preceding paragraphs we have called attention chiefly to the heating which bodies may experience because mechanical energy is expended upon them. But common experience teaches us that a body may also be heated by bringing it into contact with a hotter body. In this case the increased velocity of the molecules is sue to energy received by collisions with the more energetically moving molecules of the hotter body. Whether the energy is received by the first method or by the second, the amount of energy which must be imparted to the molecules of one gram of water to raise it through 1^{circ}C must evidently always be the same, viz. 427 gram meters, or 42,000,000 ergs. This energy is called *heat energy* as soon and only as soon as it exists in the form of molecular vibrations. If it exists in the form of a moving or lifted mass of compressed spring, it is called simply mechanical energy. Hence, in the first method of heating, the heat energy is *created* at the expense of mechanical energy; in the second method the heat energy is simply *transferred* from one body to another. *A calorie may now be defined, without reference to water or any other particular substance, simply as 42,000,000 ergs of heat energy.*

9.4.2 Definition of specific heat

tion_of_specific_heat

When we experiment upon different substances we find that it requires wholly different amounts of heat energy to produce in one gram of mass one degree of change in temperature.

Let 100 g of lead shot be placed in one test tube, 100 g of bits of iron wire in another, and 100 g of aluminum wire in a third. Let them all be placed in a pail of boiling water for ten or fifteen minutes, care being taken not to allow any of the water to enter any of the tubes. Let three small vessels be provided, each of which contains 100 g of water at the temperature of the room. Let the heated shot be poured into the first beaker, and after thorough stirring let the rise in the tmperature of the water be noted. Let the same be done with the other metals. The aluminum will be found to raise the temperatuere about twice as much as the iron, and the iron about three times as much as the lead. Hence, since the three metals have cooled through apporcimately the same number of degrees, we must conclude that about six times as much heat has passed out ot the aluminum as out of the lead; i.e. each gram of aluminus in cooling $1°$C gives out about six times as many calories as a gram of lead.

The number of calories taken up by ont gram of a substance when its temperature rises through $1°$*, or given up when it falls through* $1°$*, is called the specific heat of that substance.*

It will be seen from this definition, and the definition of the calorie, that the specific heat of water is 1.

9.4.3 Determination of specific heat by the method of mixtures

The preceding experiments illustrate a method for measuring accurately the specific heats of different substances. For, in accordance with the principle of the conservation of energy, when hot and cold bodies are mixed, as in these experiments, so that heat energy passes from ont to the other, *the gain in the heat energy on one must be just equal to the loss in the heat energy of the other.*

This method is by far the most common one for determining the specific heats of substances. It is known as the *method of mixtures.*

Suppose, to take an actual case, that the initial temperature of the shot used in section Definition_of_specific_heat 9.4.2 was 95°C, and that of the water 19.7°C, and that, after mixing, the temperature of teh water and shot was 22°C. Then, since 100 g of water has had its temperature raised through $22° - 19.7° = 2.3°$C, we know that 230 calories of heat have entered the water. Since the temperature of the shot fell through $95° - 22° = 73°$, the number of calories given up by the 100 g of shot in falling 1°C was $\frac{230}{73} = 3.15$. hence the specific heat of lead, i.e. *the number* of calories of heat given up by 1 g of lead when its temperature falls 1°C, is $\frac{3.15}{100} = 0.0315$.

Or again, we may work out the problem algebraically as follows. Let x equal the specific heat of lead. Then the number of calories which come out of the shot is $100 \cdot (95 - 22)x$, and the number which enter the water is $100 \cdot (22 - 19.7)$. Since, then, the heat lost by the shot is equal to the heat gained by the water, we have

$$100 \cdot (95 - 22)x = 100 \cdot (22 - 19.7), \text{ or } x = 0.0315$$

.

By experiments of this sort the specific heats of some of the common substances have been found to be as follows. Add the table from p. 185

9.4.4 Questions and problems

9.5 Heat Engines

9.5.1 The modern steam engine

Thus far we ahve considered only cases in which mechanical energy was transformed into heat energy. In all heat engines we have examples of exactly the reverse operation, namely the transformation ofheat energy back into mechanical energy. How this is done may best be understood from the surdy of various modern forms of heat engines. The invention of the form of the steam engine which is now in use is due to james Watt, who, at the time of the invention (1768), was an instrument maker in the University of Glasqlow.

The operation of such a machine can best be understood from the ideal diagram shoen in Fig. 188. Steam generated by the fire F in the boiler B passes through the pipe S into the steam chest V, and thence through the passage N into the cylinder C where its pressure forces the piston P to the left. It will be seen from the figure that, as the driving rod R moves toward the left, the so-called eccentric rod R', which controls the valve V, moves toward the right. Hence, when the piston has reached the left end of its stroke the passage N will have been closed, while the passage M will have been opened, thus throwing the pressure from the lsft to the right side of the piston, and at the same time putting the right end of the cylinder, which is full of spent steam, in the to connection with the exhaust pipe E. This operation goes on continually, the rod R' opening and closing the passages M and N at just the proper moments to keep the piston moving back and forth throughout the elngth of the cylinder. The shaft carries a heavy fly wheel W, the great inertial of which insures constancy in speed. The motion of the shaft is communicated to any desired machinery by means of a belt which passes over the pulley W'.

9.5.2 Condensing and noncondensing engines

In most stationary engines the exhaust E leads to the condenser which consists of a chamber Q, into which plays a jet of cold water T, and in which a partial vaccum is maintained by means of an air pump. In the best engines the pressure within Q is not more than 3 to 5 cm of mercury, i.e. not more than a pound to the square inch. Hence the condenser reduces the back pressure against that end of the piston which is open to the atmosphere from 15 lb down to 1 lb, and thus increases the effective pressure which the steam on the other side of the piston can exert. Since, however, the addition of the condenser makes the engine more expensive, more heave, and mote complicated, it is generally omitted on locomotives, and on other engines in which simplicity, compactness, and stability are of more importance than economy of guel. It is obvious that if a noncondensing engine is to have the same effective pressure on the piston head as a condensing engine, the pressure maintained within the boiler must be about 15 lb higher. For this reason non condensing engines are often called high-pressure engines. Such engines can easily be recognized by the puffs of exhaust steam which they send out into the atmosphere at each stroke of the piston.

9.5.3 The eccentric

In practice the valve rod R' is not attached as in the ideal engine indicated in Fig. 188, but motion is communicated to it by a so-called *eccentric*. This consists of a circular disk K (Fig. 189) rigidly attached to the axle, but so set that its center does not coincide with the center of the axle A. The disk K rotates inside the collar C and thus communicates to the eccentric rod R' a back-and-forth motion which operates the valve V in such a way as to have the fire in contact with as large a surface as possible. In the tubular boiler this end is accomplished by causing the flames to pass through a large number of metal tubes in water. The arrangment of the furnace and the boiler may be seen from the diagram of a locolotive shown in Fig. 190.

Governor

9.5.4 The draft

In order to suck the flames through the tubes B of the boiler a powerful draft is required. In locomotives this is obtained by running the exhaust steam from the cylinder C (Fig. 190) into the smockstack E through the blower F. The strong current thgough F draws with it a portion of the air from the smoke box D, thus procducing within D a partial vacuum into which a powerful draft rushes from the furnace through the tubes B. The coal consumption of an ordinary locomotive is from one-fourth ton to one ton per hour.

In stationary engines a draft if obtained by making the smokestack very high. since in this case th epressure which is forcing the air through the furnace is equal to the difference in the weights of columns of air of unit cross section inside and outside the chimney, it is evident that this pressure will be greater the greater the height of the smoke stack. This is the reason for the immense heights given to chimneys in large power plants.

9.5.5 The governor

Fig. 191 shows an ingenious device of Watt's, called a *governor*, for regulating automatically the speed with which a stationay engine runs. If it runs too fast, the heavy rotating balls B move apart and upward, and in so soing operate a valve which partially shuts the supply of steam from the cylinder.

9.5.6 The reversing and speed-regulating device of a locomotive

In order to control the speed of a locomotive and reverse it when desired, two eccentrics, A, A, (Fig. 102), are provided. These are set exactly opposite on the shaft, so that the two ponts B and B are always moving in opposite directions, while the point midway between them remains stationay. In the position shown in the figure the motion of the valve rod T is controlled entirely by motion of the point B, but when the lever is thrown to the left the point B' moves up and assumes complete control of T. Since the two eccentrics are set oppositely, throwing the lever reverses instantly the direction in which steam acts against the piston head and thus reverses the locomotive. If the lever is set in the middle, all communication between the vlave chest and the steam chest is cut off. The speed may be controlled at will by setting the lever at intermediate positions, for in this way steam passages may be opened as much or as little as desired.

9.5.7 Compound engines

In an engine which has but a single cylinder the full force of the steam has not been spent when the cylinder is opened to the exhaust. The waste of energy which this entails is obviated in the compound egine by allowing the partially stent steam to pass into a second cylinder of larger area than the first.

The most effecient of modern engines have three and sometimes four cylinders of this sort, and the engines are addordingly called emphtriple or *quadruple expansion engines*. Fig. 193 shows the realstion between any two successive cylinders from the eccentric, valves C^1, D^2, E^3 open simultaneously and thus permit steam from the boiler to enter the small cylinder A, while the partially spent steam in the other end of the same cylinder passes through D^2 into B, and the more fully exhausted steam in the upper end of B passes out through E^2. At the upper end of the stroke of the pistons P and P', C^1, D^2, and E^2 automatically close, while C^2, D^1, and E^1 simultaneously open and thus reversy the direction of motion of both pistons. These pistons are attached to the same shaft.

9.5.8 Efficiency of a steam engine

We have seen that it is possible to transform completely a geven amount of mechanical energy into heat energy. This is done whenever a moving body is brought to rest by means of a frictional resistance. But the inverse operation, namely that of transforming heat energy into mechanical energy, differs in this respedt, that it is only a comparatively small fraction of the heat developed by combustion which can be transformed into work. For it is not difficult to see that in every steam engine at least a part of the heat must of necessity pass over with the exhaust steam into the condenser or out into the atmosphere. This loss is so great that even in an ideal engine not more than about 23% of the heat of combustion could be transformed into work. In practice the very best condensing engines of the quadruple expansion type transform into mechanical work not more than 17% of the heat of compustion. Ordinary locomotives utilize at most not more than 8%. *The efficiency of a heat engine is defined as the ratio between the heat utilized, or transformed into work, and the total heat expended.* The efficiency of the best steam engines is therefore about $\frac{17}{23}$ or 75% of that of an ideal heat engine, while that of the ordinary locomotive is only about $\frac{6}{23}$ or 26% of the ideal limit.

9.5.9 The principle of the gas engine

Within the last decade gas engines have begun to replace steam engines to a very greate extent, especially for small power purposes. These engines are driven by properly timed explosions of a mixture of gas and air occurring within the cylinder.

Fig. 194 is a diagram illustrating the four stages into which it is convenient to divide the complete cycle of operations which goes on within such an engine. Suppose that the heavy fly wheels W have already been set in motion. As the piston p moves to the right in the first stroke (see 1) the valve E opens and an explosive mixture of gas and air is drawn into the cylinder through E. As the piston returns to the left (see 2) valve E closes, and the mixture of gas and air is compressed into a small space in the left end of the cylinder. An electric spark ignites the explosive mixture, and the force of the explosion drives the piston violently to the right (see 3). At the beginning of teh return stroke (see 4) the exhaust valve D opens, and as the piston moves to the left the spent gaseous products of the explosion are forced out of the cylinder. The initial condition is thus restored and the cycle begins over again.

Since it is only during the third stroke that the engine is receiving energy from the exploding gas, the fly wheel is always made very heavy so that the energy stored up in it in the third stroke may keep the machine running with little loss of speed during the other three parts of the cycle.

9.5.10 Mechanism of the gas engine

The mechanism by which the above operations are carried out in one type of modern gas engins (the Foos) may be seen from a study of Fig. 195a and 195b. 195a is a section of the left end of the engine shown in perspective in 195a. Suppose that the fly wheels W are first set in motion by hand. When the cam or eccentric c_1 (195a) drives the rod to th left it opens a valve in F through which a gas passes from the inlet pipe A into the mixing chamber I (195a and b). Here it mixes with air which entered through the pipe B. As soon as the cam c_2 has moved about to the position in which it throws the lever arm l_1 to the left, the rod G_1 is forced upward and the inlet valve E (195b) is therefore opened. This happens at the beginning of the first stage (section 3.5) when the piston K is beginning to move to the right. Hence the explosive mixture is at once drawn into C (195b). At the beginning of the third stage a third eccentric rod N operated by an eccentric c_4 (195a), and thus produces a spark which explodes the gas. At the beginning of the fourth stage the cam c_3 drives the lever arm l_2 (195a) to the left, and thus with the aid of G_2 (195a and b) opens the exhaust valve D (195b) and thus permits the spent gases to escape. This completes the cycle.

Sinve each of the four cams, $c_1.c_2, c_3, c_4$, must open its valve once in two revolutions of the fly wheel, all four of these cams are placed not on the main shaft H, but on the axel of the gear wheel, M, which has twice as many teeth as has the gear wheel n on the main shaft. M therefore revolves byt once while the main shaft is revolving twice. In order that the cylinder may be kept cool it is surrounded by a jacket U through which water is kept continually circulating.

The efficiency of the gas engine is often as high at 25%, or nearly double that of the best steam engines. Furthermore, it is free from smoke, is very compact, and may be started at a moment's notice. On the other hand, the fule, gas of gasoline, is comparatively expensive. Most automobiles are run by gasoline engines, chiefly because the lightness of the engines and of the fule to be carried are here considerations of great importance.

9.5.11 The steam turbine

The steam turbine represents the latest development of the heat engine. In principle it id very much like the common windmill, the ciief defference being that it is steam instead of air which is driven at a high velocity against a series of blades which are arranged radially about the circumference of the wheel which is to be set into rotation. The steam however, unlike the wind, is always directed byt noxxles (*A* and *B*, Fig. 196a), at the angle of greatest efficiency against the blades. Furthermore, since the energy of the steam is not nearly spent after it has passed through one set of blades, such as that shown in Fig. 196a, it is in practice always passed through a whole series of such sets (Fig. 196b), every alternate row of which is ridgidly attached to the rotating shaft, while the intermediate rows are fastened to the immovable outer jacket of the engine, and only serve as guides to redirect the steam at the most favorable angle against the next row of movble blades. In this way the steam is kept alternately bounding from fixed to movable blades till its energy is expended. The number of rows of blades is often as high as sixteen.

Turbines are at present coming rapidly into use, chiefly for large power purposes. Their advantages over the reciprocating steam engine lie first in the fact that they run with almost no jarring and thererfore require much lighter and less expensive foundations, and second in the fact that they occupy less than one tehnth the floor space of ord inary engines of the same capacity. Their efficiency is fully as high as that of the best reciprocating engines. The highest speeds attained by vessels at sea level, namely about 40 mph, have been made with the aid of steam turbines. The largest vessel which has thus far ever been launched, the thirty-thousand-ton Cunard steamer *Carmania*, which made her maiden trip in December, 1905, is driven by three steam turbines, which have a total of no less than 1,250,000 blades.

9.5.12 Questions and problems

Add this section.

Chapter 10

Change of State

10.1 Fusion

[1]

10.1.1 Heat of fusion

If on a cold day in winter a quantity of now is brought in from out of doors, where the temperature is below 0°C, and placed over a source of heat, a thermometer plunged into the snow will be found to rise slowly until the temperature reaches c°C, when it will become stationary and remain so during all the time that the snow is melting, provided only that the contents of the vessel are continuously and vigorously stirred. As soon as the snow is all melted the temperature will begin to rise again.

Since the temperature of ice at 0°C is the same as the temperature of water at °C, it is evident from this experiment that when ice is being changed to water the entrance of heat energy into it does no produce any change in the average kinetic energy of its molecules. This energy must therefore all be expended in pulling apart the molecules of the crystals of which the ice is composed and thus reducing it to a form in which the molecules are held together less intimately, i.e. to the liquid form. In other words, the energy which existed in the flame as the kinetic energy of molecular motion has been transformed, upon passage into the melting solid, into the potential energy of molecules which have been pulled apart against the force of their mutual attraction. *The number of calories of heat energy required to melt one gram of any substance without producing any change in its temperature is called the heat of fusion of that substance.*

[1]This subject should be preceded by a laboratory exercise on the curve of cooling through the point of fusion, and following by the dtermination of the heat of fusion of ice. See, for example, Experiments 21 and 22 of the authors' manual.

10.1.2 Numerical value of heat of fusion

Since it is found to require about 80 times as long for a given flame to melt a quantity of snow as to raise th emelted snow through 1°C, we conclude that it requires about 80 calories of heat to melt 1 g of snow or ice. This constant is, nowever, much more accurately determined by the method of mixtures. Thus suppose that a piece of ice weighing, for example, 131 g is dropped into 500 g of water at 40°C, and suppose that after the ice is all melted the temperature of the mixture is found to be 15°C. The number of calories which have some out of the water is $500 \cdot (40 - 15) = 12,500$. But $131 \cdot 15 = 1965$ calories of this heat must have been used in raising the ice from 0°C to 15°C after it, by melting, became water at 0°C. The remainder of the heat, namely $12,500 - 1965 = 10,535$, must have been used in melting the 500 g of ice. Hence the number of calories required to melt 1 g of ice is $\frac{10535}{131} = 80.4$.

To state the problem algebraically, let $x =$ the heat of fusion of ice. Then we have

$$1313x + 1965 = 12,500; \ mathrmi.e. \ x = 80.4$$

According to them most careful determinations the heat of fusion of ice is 80.8 calories.

10.1.3 Heat given out when water freezes

Let snow and salt be added to a beaker of water until the temperature of the liquid mixture is as low at −10°C or $^{-12\circ}$C. Then let a test tube containing a thermometer and a quantity of pure water be thrust into the cold solution. If ther thermometer is kept very quiet, the temperature of the water in the test tube will fall four of five, or even then, degrees below 0°C without producing solidification. But as soon as the thermometer is stirred, or even a small crystal of ice is dropped into the neck of the tube, the ice crystals will form with great suddenness and at the same time the thermometer will rise to 0°C where it will remain until all the water is frozen.

The experiment shows in a very striking way that the process of freezing is a heat-evolving process. This was to have been expected from the principle of the conservation of energy; for since it takes 80 calories of heat energy to turn a gram of ice at 0°C into water at 0°C, this amount of energy must be reappear when the water turns back into ice.

10.1.4 Utilization of heat evolved in freezing

The heat given off by the freezing of water is often turned to practical account; e.g. tubes of water are sometimes placed in vegetable cellers to prevent the vegetables from freezing. The effectiveness of this procedure is due to the fact tha the temperature at which the vegetables freeze is slightly lower than 0°C. As the temperature in the celler falls the water there fore begine to freeze first, and in so doing evolves enough heat to prevent the temperature of the room from falling as far below 0°C as it otherwise would.

It is partly because of the heat evolved by the freexing of large bodies of water that the temperature never falls so low in the vicinity of large lakes as it does in inland localities.

10.1.5 Latent heat

Before the time of Joule, when heat was supposed to be a weightless fluis, the heat which disappears in a substance when it melts and reappears again when it solidifies was called *latent* or *hidden* heat. Thus water was said to have a latent heat of 80 calories. This expression is still in common use, although with the change which has taken place in our views of the nature of heat, its appropriateness is entirely gone. For the heat energy which is required to change a substance from a solid to a liquid does not exist within the liquid as condealed or hidden heat energy, but was instead *ceased to exist as heat energy at all*, having been transformed into the potential energy of partially seperated molecules, i.e. it represents the work which has been done in effecting the change of state.

10.1.6 Melting points of crystalline substances

If a piece of ice is placed in a vessel of boiling water for an instant and then removed and wiped, it will not be found to be in the slightest degree warmer than a piece of ice which has not been exposed to the heat of warm water. The melting point of ice is, therefore, a perfectly fixed, definite temperature, above which the ice can never be raised so long as it remains ice, no matter mow fast heat is applied to it. All crystalline substances are found to behave exactly like ice in this respect, each substance of this class habing its characteristic melting point. The following table gives the melting points of some of the commoner crystalline substances.

Add this table, page 199.

We may summerize the experiments upon melting points of crystalline substances in two follow laws.

1. *The temperatures of solidification and of fusion are the same.*
2. *The temperature of the melting or solidifying substance remains constant from the moment at which melting or solidification begins until the process is completed.*

10.1.7 Fusion of noncrystalline or amorphous substances

Let the end of a glass rod be held in a Bunsen flame. Instead of changing suddenly from the solid to the liquid state, it will gradually from softer and softer until, if the flame is sufficiently not, a drop of molten glass will fal from the end of the rod.

jIf the temperature of the rod had been measured during this process, it would have beenfound to be continually rising. This behavior, so completely unlike that of crystalline substances, is characteristic of tar, way, resin, glue, gutta-percha, alcohol, carbon, and in general of all amorphous substances. Such substances cannot be said to have any definite melting points at all, for they apss through all stages of viscosity both in melting and in solidifying. It is in virtue of this property that glas and other similar substances can be heated to softness and then molded into any desired shapes.

10.1.8 Change of volume on solidifying

One has only to reflect that ice floats, or that bottles of crocks of water hurst when they freeze, in order to know that water expands upon solidifying. In fact, 1 ft^3 of water becomes 1.09 ^{3} of ice, thus expanding more than on twelveth of its initial volume when it freezes. This may seem strange in view of the fact that the molecules are certainly more closely knit together in the solid than in the liquid state; but he strangeness disappears when we reflect that in freezing the molecules of water group themselves into crystals, and that this operation presumable leaves comparatively free spaces between different crystals, so that, slthough groups of individual molecules are more closely joined than before, the total volume occupid by the whole assemblage of molecules is greater.

But the great majority of crystalline substances are unlike water in the respect, for, with the exception of antimony and bismuth, they all contract upon solidifying and expand on liquifying. It is only from substances which expand, or which, like cast iron, change in volume very little on solidifying, that sharp casting can be made. For it is clear that contracting substances cannot retain the shape of the mold. It is for this reason that cold and silver coins must be stamped rather tahncast. Any metal from which type is to be cast must be one which expands upon solidifying, for it need scarcely be said that perfectly sharp outlines are indispensalbe to good type. Ordinary type metal is an allow of lead, antimony, and copper which fulfills these requirements.

10.1.9 Effect of the expansion which water undergoes on freezing

If water were not unlike most substances in that it expands on freexing, many, if not all, of the forms of life which exist on the earth would be impossible. For in winter the ice would sink on ponds and lakes as fast as it froze, and soon our rivers, lakes, and perhaps our oceans also would become solid ice.

The force exerted by the expansion of freezing water is very great. Steel bonbs have been burst by filling them with water and exposing them on cold winter nights. One of the chief agants in the disintegration of rocks is the freezing and consequent expansion of water which has percolated into them.

10.1.10 Pressure lowers the melting point of substances which expand on solidifying

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