Bug found in GLPK

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March 2010

I have encountered a bug while using the 4.34 version of GLPK, used with Matlab (with GLPKmex), for Windows.

In the function where the bug appeared, I first build the instance of the system I want to solve (A, b, c, vartype, ctype, sense) and I then call glpk on the instance. Most of the time, the correct answer seems to be returned. However, I have found several examples on which the solution returned by GLPK doesn't satisfy the constraints at all, even though it tells me that he found the optimal solution (status 5). Note that the particular problem I am exposing hereunder is of completely reasonable size (9 constraints and 2 continous variables).

I have tried to solve exactly the same problem with an other solver for Matlab (SeDuMi) and this one gives me the right answer. Yet, I would really prefer to use the GLPK solver as it is much more efficient for the problems I have to solve and it enables me to solve problems of much higher size. That's why it would really help me in my research if you could help me solve this bug.

Hereunder, you will find more precisions about a particular example on which I had a problem as well as the Matlab source code and the error logs.

1 The problematic example

The problem is the following :

$$\begin{aligned} \min \quad & c^T x \\ \text{subject to} \quad & \tilde{A}x \leq \tilde{b} \end{aligned}$$

where

$$\tilde{A} = \begin{pmatrix} 1 & -2/3 \\ 1/6 & 1 \\ -5/6 & -1 \\ -2/3 & 1/3 \\ 0 & -4/3 \\ 1 & 2/3 \\ 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \tilde{b} = \begin{pmatrix} 1/3 \\ 1/2 \\ -1/2 \\ 0 \\ -1/3 \\ 2/3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \qquad c = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

I also use the parameters

$$\text{ctype} = \begin{pmatrix} U \\ U \\ U \\ U \\ U \\ U \\ U \\ U \\ U \end{pmatrix}, \qquad \text{vartype} = \begin{pmatrix} C \\ C \end{pmatrix}$$

I have tried that exact problem with GLPK and it works. However, in my case, when I construct the matrix $\tilde{A}$ and the vector $\tilde{b}$ automatically with my function, I commit some small numerical errors ($\sim 10^{-15}$). The matrix A and the vector b I am using are the following :

$$A = \tilde{A} + \begin{pmatrix} 0 & -0.111022302462516 \\ 0.027755575615629 & 0 \\ 0.111022302462516 & 0 \\ -0.111022302462516 & -0.055511151231258 \\ 0.055511151231258 & 0 \\ 0 & 0.111022302462516 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \cdot 10^{-15}$$

$$b = \tilde{b} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.055511151231258 \\ 0.111022302462516 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot 10^{-15}$$

Then I call the glpk function like this :

```
x = glpk(c,A,b,[],[],ctype, vartype, -1);
```

And here is the result :

$$Ax = \begin{pmatrix} 1.333333333333333 \\ 0.500000000000000 \\ -1.500000000000000 \\ -0.916666666666666 \\ -0.333333333333333 \\ 1.666666666666666 \\ 1.750000000000000 \\ -1.500000000000000 \\ -0.250000000000000 \end{pmatrix} \overset{?}{\leq} \begin{pmatrix} 0.333333333333333 \\ 0.500000000000000 \\ -0.500000000000000 \\ 0 \\ -0.333333333333333 \\ 0.666666666666667 \\ 1.000000000000000 \\ 0 \\ 0 \end{pmatrix} = b$$

As you can see, in this example, three constraints are violated and I can't explain why. The perturbation on my system should be too small to generate such a violation. Hereunder, I copy you the Matlab code I used to generate the example, so that you can eventually check it by yourself. You can find the Matlab `PRIntervalMatrices` function as well as the following test attached with this report.

2 Matlab code

```
% Common data
E = ones(3);
F = [0 0 0 ; 0 1 1 ; 1 0 0];

n = 3;

c = [1 0]';
ctype   = char('U'*ones(3*n, 1));
vartype = char('C'*ones(n-1, 1));

fprintf('-----------------------------------------------------\n')
fprintf('Case given by my function \n')
fprintf('-----------------------------------------------------\n')

[PRmin, PRmax, AIM, bIM] = PRIntervalMatrices(E, F);

fprintf('The matrix A \n\n')
disp(AIM)
fprintf('The vector b \n\n')
disp(bIM)

[x, f, status] = glpk(c,AIM,bIM,[],[],ctype, vartype, -1);
fprintf('A*x \n\n')
disp(AIM*x)
fprintf('b \n\n')
disp(bIM)
fprintf('A*x-b \n\n')
disp(AIM*x-bIM)
fprintf('status \n\n')
disp(status)


fprintf('-----------------------------------------------------\n')
fprintf('Case with hardcoded matrix \\tilde{A} \n')
fprintf('-----------------------------------------------------\n')

A = [1 -2/3 ; 1/6 1 ; -5/6 -1 ; -2/3 1/3 ; 0 -4/3 ; 1 2/3 ; 1 1 ; -1 0 ; 0 -1];
b = [1/3 1/2 -1/2 0 -1/3 2/3 1 0 0]';

fprintf('The matrix \\tilde{A} \n\n')
```

```
disp(A)
fprintf('The vector \\tilde{b} \n\n')
disp(b)

[x, f, status] = glpk(c,A,b,[],[],ctype, vartype, -1);
fprintf('\\tilde{A}*x \n\n')
disp(AIM*x)
fprintf('\\tilde{b} \n\n')
disp(bIM)
fprintf('\\tilde{A}*x-\\tilde{b} \n\n')
disp(AIM*x-bIM)
fprintf('status \n\n')
disp(status)


fprintf('-------------------------------------------------------\n')
fprintf('Difference between A and \\tilde{A} and b and \\tilde{b} \n')
fprintf('-------------------------------------------------------\n')

fprintf('A - \\tilde{A} \n\n')
disp(A-AIM)
fprintf('b - \\tilde{b} \n\n')
disp(b - bIM)
```

3 Matlab's result in console

```
-------------------------------------------------------
Case given by my function
-------------------------------------------------------
The matrix A

   1.000000000000000  -0.666666666666667
   0.166666666666667   1.000000000000000
  -0.833333333333333  -1.000000000000000
  -0.666666666666667   0.333333333333333
   0.000000000000000  -1.333333333333333
   1.000000000000000   0.666666666666667
   1.000000000000000   1.000000000000000
  -1.000000000000000                   0
                   0  -1.000000000000000
```

```
The vector b

   0.333333333333333
   0.500000000000000
  -0.500000000000000
                   0
  -0.333333333333333
   0.666666666666667
   1.000000000000000
                   0
                   0

A*x

   1.333333333333333
   0.500000000000000
  -1.500000000000000
  -0.916666666666666
  -0.333333333333333
   1.666666666666666
   1.750000000000000
  -1.500000000000000
  -0.250000000000000

b

   0.333333333333333
   0.500000000000000
  -0.500000000000000
                   0
  -0.333333333333333
   0.666666666666667
   1.000000000000000
                   0
                   0

A*x-b

   1.000000000000000
                   0
  -1.000000000000000
  -0.916666666666666
  -0.000000000000000
   1.000000000000000
   0.750000000000000
  -1.500000000000000
  -0.250000000000000
```

```
status

     5

-----------------------------------------------------
Case with hardcoded matrix \tilde{A}
-----------------------------------------------------
The matrix \tilde{A}

   1.000000000000000  -0.666666666666667
   0.166666666666667   1.000000000000000
  -0.833333333333333  -1.000000000000000
  -0.666666666666667   0.333333333333333
                   0  -1.333333333333333
   1.000000000000000   0.666666666666667
   1.000000000000000   1.000000000000000
  -1.000000000000000                   0
                   0  -1.000000000000000

The vector \tilde{b}

   0.333333333333333
   0.500000000000000
  -0.500000000000000
                   0
  -0.333333333333333
   0.666666666666667
   1.000000000000000
                   0
                   0

\tilde{A}*x

   0.333333333333333
   0.333333333333333
  -0.666666666666667
  -0.250000000000000
  -0.333333333333333
   0.666666666666667
   0.750000000000000
  -0.500000000000000
  -0.250000000000000
```

```
\tilde{b}

   0.333333333333333
   0.500000000000000
  -0.500000000000000
                   0
  -0.333333333333333
   0.666666666666667
   1.000000000000000
                   0
                   0

\tilde{A}*x-\tilde{b}

                   0
  -0.166666666666667
  -0.166666666666667
  -0.250000000000000
                   0
                   0
  -0.250000000000000
  -0.500000000000000
  -0.250000000000000

status

     5

-------------------------------------------------------
Difference between A and \tilde{A} and b and \tilde{b}
-------------------------------------------------------
A - \tilde{A}

  1.0e-015 *

                   0   0.111022302462516
  -0.027755575615629                   0
  -0.111022302462516                   0
   0.111022302462516   0.055511151231258
  -0.055511151231258                   0
                   0  -0.111022302462516
                   0                   0
                   0                   0
                   0                   0
```

```
b - \tilde{b}

  1.0e-015 *

                    0
                    0
                    0
                    0
  -0.055511151231258
  -0.111022302462516
                    0
                    0
                    0
```