

Bar (or Truss) problem

For FEM

Strong form

$$\frac{d\sigma_{11}}{dx_1} + \rho g_1 = 0$$

Weak form

Using test function vectors $\hat{\mathbf{u}}$ in a line,

$$\int_{x_1} A \frac{d\sigma_{11}}{dx_1} \hat{u}_1 dx_1 + \int_{x_1} A \rho g_1 \hat{u}_1 dx_1 = 0$$

where

$$A = \int_{x_3} \int_{x_2} dx_2 dx_3$$

Similar to the general case of linear elasticity, we obtain

$$\int_{x_1} A \sigma_{11} \frac{d\hat{u}_1}{dx_1} dx_1 = \int_S A t_1 \hat{u}_1 dS + \int_{x_1} A \rho g_1 \hat{u}_1 dx_1$$

For GetFEM++

The term

$$\int_{x_1} A \sigma_{11} \frac{d\hat{u}_1}{dx_1} dx_1 \rightarrow "A * E * \text{Grad}_u(1,1) * \text{Grad_Test}_u(1,1)"$$

because,

$$\sigma_{11} \rightarrow \sigma_{11} = E \frac{du_1}{dx_1} \rightarrow "E * \text{Grad}_u(1,1)"$$

$$\frac{d\hat{u}_1}{dx_1} \rightarrow " \text{Grad_Test}_u(1,1) "$$