knowing the patient orientation in the OR, i.e., the direction of gravity with respect to the brain can be challenging. For example, in frameless stereotactic procedures, the reference emitter is commonly attached to the patient's fixation. This allows tracked instruments to be directly related to the patient's image volume once the patient has been registered. This has the advantage that as the patient's bed is lowered and/or rotated, the reference frame is rotated with the patient. However, in so doing, the absolute reference to the OR (the reference frame of gravity) can be lost unless a second reference emitter is attached to OR space (not commonly done). Without a second reference emitter, the direction of gravity relative to the patient is lost. One approach to addressing this uncertainty is to generate an atlas of deformation solutions based on a range of possible surgical presentations. This has the added benefit to efficiency by allowing for precomputation of the deformation atlas.

In this paper, a realization to the brain shift compensation problem is proposed using a precomputed deformation atlas. Operationally, Eqs. (1) and (2) are solved for a range of possible factors causing brain shift. Let the deformation atlas, **E**, be the matrix obtained by assembling these model solutions whereby **E** is of size $(n \times 3) \times m$, where n is the number of nodes in the finite element mesh, 3 is the number of Cartesian displacement components at each node, and m the number of model solutions. In general, $n \times 3$ is significantly larger than m, so **E** is a rectangular matrix. The model-data misfit error between a linear combination of precomputed displacement solutions and the actual displacements can be written as

$$\varepsilon_{\text{volume}} = [\mathbf{E}]\{\alpha\} - \{U\} \tag{3}$$

where U is the measured volumetric intraoperative shift, i.e., shift at all nodes and is $(n \times 3) \times 1$ vector, and α is the $m \times 1$ vector of regression coefficients. This can then be expressed as the least squared error objective function,

$$G_{\text{volume}}(\alpha) = ([\mathbf{E}]\{\alpha\} - \{U\})^{\mathrm{T}}([\mathbf{E}]\{\alpha\} - \{U\}) \tag{4}$$

As noted above, the measurements U are often incomplete or sparse. As a result, model solutions within **E** are interpolated to the specific measured intraoperative data points and these interpolated solutions are assembled in an intraoperative sparse deformation atlas, **M**. Thus **M** is of size $(n_s \times 3) \times m$, where n_s is the number of points for which sparse intraoperative data has been measured. The displacement data sets in **M** serve as the training samples for the inverse model and reduce the model-data misfit error, and objective function to

$$\varepsilon_{\text{sparse}} = [\mathbf{M}]\{\alpha\} - \{u\} \tag{5}$$

$$G_{\text{sparse}}(\alpha) = ([\mathbf{M}]\{\alpha\} - \{u\})^{\mathrm{T}}([\mathbf{M}]\{\alpha\} - \{u\}), \tag{6}$$

respectively. Here, u is the sparse intraoperative shift measured at n_s points in the brain. This, however, can transform the problem into an undetermined system because there are usually more regression coefficients than measurement points (i.e. $m > n_s$). While minimum norm solutions can produce perfect fitting of the data they are often unsatisfying with respect to volumetric shift prediction due to the measurements being confined to a small spatial region (e.g. craniotomy in this case). This is addressed by introducing an extra constraint, which has the effect of encouraging a spatially smooth displacement field that is confined within the cranial extents. The modified objective function can be written as,

$$G_{\text{sparse}}(\alpha) = ([\mathbf{M}]\{\alpha\} - \{u\})^{\mathrm{T}}([\mathbf{M}]\{\alpha\} - \{u\}) + \phi[W]^{\mathrm{T}}\{\Upsilon\}\{\alpha\} \tag{7}$$

The second term in this expression is a function of the mechanical strain energy at each point within the model and serves to constrain the regression coefficients to values that would also minimize the elastic energy across the deformation atlas. In this expression, the term Υ refers to the linear elastic strain energy matrix, described by $\Upsilon_{i,j} = 1/2\{\epsilon_{i,j}\}^t[S_{i,j}]\{\epsilon_{i,j}\}$, where $S_{i,j}$, $\epsilon_{i,j}$ is the elastic stiffness tensor, and Cartesian strain tensor in vector form, respectively, for the ith node of the jth solution from the atlas (material properties are in Appendix A.2). With the development of any multi-term objective function (Eq. (7)), care must be taken to allow proper scaling of terms such that the data is matched optimally while also retaining the beneficial effects of constraints. This process of regularization is often problem specific. With this in mind, a distance based weighting factor vector $W^{\mathrm{T}} = [W_1, W_2, W_3, \ldots]$ is introduced that is similar to that in Lynch (2004), and is used with the strain energy matrix described above. The weighting vector is constructed as,

$$W_i = \frac{1}{(1 + r_i/l)\mathrm{e}^{-r_i/l}} \tag{8}$$

where r_i is the distance between the centroid of the measurement nodes and the ith node in the brain volume. The l is a characteristic length that specifies the domain over which measurement nodes should have influence. With that, the form of Eq. (8) reduces the strain energy constrain within the region of measurements nodes, i.e. the craniotomy in this case. While displacements tend to be small in areas remote from the craniotomy, they will have increased strain energy and increased weighting. When Eq. (7) is optimized for the regression coefficients, the net effect of the constraint term is to enforce a minimal elastic energy state on remote regions of the domain while selecting coefficients that best match the shift in the cranial and tumor regions. ϕ in Eq. (7) provides a scaling role such that the solution is not biased by the strain energy constraint term. The values for l and ϕ were found empirically and are 0.125 and 1/2700, respectively.

Finally, setting the partial derivative to zero, the optimum for Eq. (7) has a direct solution for $\{\alpha\}$. Once the regression coefficients are determined, these are used to calculate the full volume displacements using

$$\{U^*\} = E\alpha \tag{9}$$