

# Schumpeterian competition with SWARM, an agent-based framework for computational modelling

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## 1 Introduction

The Nelson and Winter 1977 model, or simply abbreviated in this document “NW77”, is a model of an industry evolution constituted only by firms according to basic principles of evolutionary economics (Schumpeterian competition) and which supports innovation and imitation by firms. The model uses probability distributions to simulate the innovative and imitative behaviours of firms and the effect of these behaviours on each firm’s productivity. The original model is described in “Dynamic Competition and Technical Progress” (Nelson and Winter 1982: 275-307), however, according to Andersen (1996), it seems that the original source code of NW77 has been lost. In an attempt to reconstruct the source code of this important model, Andersen (1996) has documented the NW77 model in a simulation environment called LSD. The objective of this paper is twofold. First, to adapt NW77 to the agent-based environment, SWARM, and compare the results with Andersen (1996) and Nelson and Winter (1982). Second, to modify the original NW77 model by relaxing the assumption in the “science-based” case that imitator firms have a global imitation horizon, i.e., if they are successful under an imitative draw, they can immediately adopt the best available productivity. Instead, in the modified model called “NWLocIm” which stands for NW model with local imitation, I consider that firms are spatially positioned on a torus surface<sup>2</sup>, in which they have limited scope for accessing the best productivity of their direct local neighbours.<sup>3</sup> The research questions are then, what is the impact of this modification by comparing the results of NWLocIm with the results of NW77? Do the evolutionary economics principles developed in Nelson and Winter (1982) give an explanation that account for these results?

## 2 NW77

NW77 implements some basic ideas about the innovation process in firms. In orthodox theory the nature of the economic problem is “to pick the best possible production and distribution, given a known set of alternatives”. However, in evolutionary theory, firms’ behaviour is conditioned by their bounded rationality (rule based decision process and inertia), choice sets are not given and the consequences of any choice are unknown. Although some choices may be clearly worse than others, there is no choice that is clearly best *ex ante* (Nelson and Winter 1982: 276). Firms need to do specific R&D investment to innovate. The competitive advantage of firms is based on their innovative capacity. As Nelson & Winter put it, quoting Schumpeter, “the returns to innovation stem from the transient monopoly of a new product or process provided by imitator lag (ibid. p.

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<sup>2</sup> A torus surface is topologically equivalent to a square with edges connected to opposite edges. For example, if a point moves off the right top edge, it reappears on the left lower edge.

<sup>3</sup> I use a Moore neighbourhood with eight neighbouring firms.

279) (in the model, only process innovations are considered). Therefore, the absence of competitors, and the ability to block imitation by competitors, is factors that in their own right influence appropriability (ibid. p. 280). The selection of firms is determined by their innovative capacity, sometimes this can be measured with the concept of “fitness”. The innovative behaviour of firms hence determines the structure of the industry and its evolution. The model aims to answer two sets of questions (ibid. p. 291): first, how does industry performance over a considerable number of periods depend on the initial concentration of the industry? Second, what are the effects of initial concentration on the way in which industry structure evolves over time? For example, when innovative R&D is not profitable (because of, e.g., poor appropriability or “easy imitation”), in what way does the survivability of firms that do innovative R&D depends on initial concentration? More generally, which initial structures tend to be stable and which unstable? Do the initially un-concentrated structures tend to concentrate over time? Do the initially concentrated structures tend to concentrate further?

Figure 1 below, presents the computational structure of NW77 as given in Andersen (1996). In the following paragraphs I will explain briefly the 15 steps of this model.

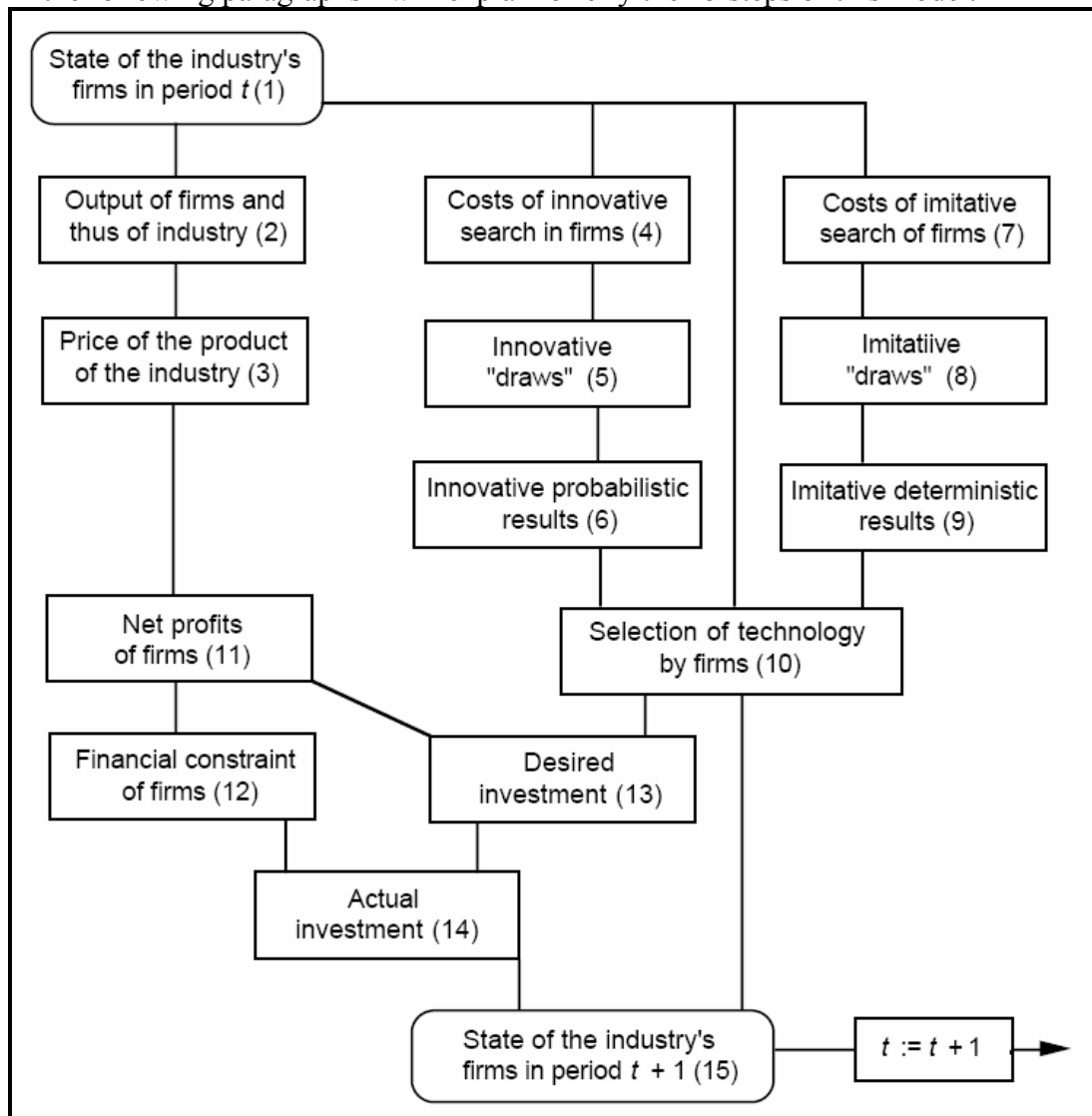


Figure 1. The computational structure of NW77. Source: Andersen (1996)

(1) The state of the industry is defined in terms of the size of the physical capital stock  $K(it)$  and the productivity of capital  $A(it)$  of each of the  $n$  firms. This state is inherited from the former period.

(2) The production is characterised by constant returns to scale. The maximum output that can be produced by a firm is  $Q(it) = A(it).K(it)$ . The actual output is equal to maximum output for each firm. Output of the industry  $Q(t)$  is found by aggregating the outputs of the  $n$  firms.

(3) The aggregate output of the industry faces exogenously given demand conditions that are characterised by unit elasticity, i.e., the same total revenue  $D$  is obtained by the industry no matter how much or little it produces. In other words, price adapts to clear the market:  $P(t) = D/Q(t)$

The next steps are describing how new technologies are found after a successful process of search or imitated and how firms' productivities are changed. Productivities are specific to individual firms  $A(it)$ . They reflect knowledge that has a fairly high degree of appropriability of the results of R&D. Technical change takes the form of process innovations and process imitations that increase the capital coefficient of individual firms  $A(it+1) \geq A(it)$ . The processes whereby new production techniques are found and productivity is changed include the following steps:

(4) The firms' costs of innovative R&D are found by fixed decision rules that determine them in proportion to the level of physical capital  $r_{in}(i).K(it)$ .

(5) The firm's chance of getting a 'draw' (an innovation) in the innovative 'lottery' ( $d_{in}$ ) is proportionate to its innovative R&D costs, and it is determined by the exogenously given character of technical change of the industry. It takes the form of a Poisson distribution with a mean number of successes per period determined by the effort of the firm as well as by the appropriability of the technology.

(6) An innovative 'draw' or success gives the firm access to another 'lottery' that determines the productivity of the innovation. This productivity depends on an exogenously given normal probability distribution. In the "science-based" case, the normal distribution has a mean  $\ln(Ascience(t))$  defined by the exponentially growing science-based state-of-the-art. The standard deviation of the distribution is fixed, and the result is transformed back from the log-form to an ordinary productivity. In the cumulative case, the normal distribution has a mean  $\ln(A(it-1))$  which depends on the productivity of firm  $i$  at the previous period.

(7) The firms' costs of imitative R&D are found by fixed decision rules that determines them in proportion to the level of physical capital  $r_{im}(i).K(it)$ . The costs are very small, since the industry comes near to pure spill-over from the innovators.

(8) Because of its imitative search effort, each firm gets access to a 'lottery'. Its probability of obtaining a 'draw', i.e. to draw a ticket from the lottery, is proportionate to its imitative search costs but is otherwise determined by exogenous factors (the difficulty of imitation in the particular industry).

(9) An imitative 'draw' or success means that the firm gets access to the best-practice technique and thus the highest productivity level obtained by any firm in the period.

(10) The attempts to improve productivity end with a comparison between the productivities obtainable by the technique inherited from the last period and the techniques which may be found by imitative and innovative search. The technique with the highest productivity is chosen. If the technique is changed, it will determine productivity of the next period (disembodied technical change). We thus have the state of technique (production routines) for period  $t + 1$ .

Now we turn to the investment decisions which determine new firms' capital:

(11) For each firm we calculate the turnover  $P(t).Q(it) = P(t).A(it).K(it)$  and then find the net profit by deducting the costs elements (which are all measured per unit of physical capital). Taken together variable production costs, capital depreciation, and interest amounts to  $c$  per unit of capital,  $c$  is assumed to be constant over all periods. The costs of innovative and imitative R&D are determined by fixed decision rules that determine them in proportion to the level of physical capital ( $r_{in}$ ,  $r_{im}$ ). Profits per unit of capital are calculated by including R&D costs as ordinary cost elements:  $p(it) = P(i)A(it) - (c + r_{in} + r_{im})$ .

(12) The maximum investment of a firm is determined by the profits of the present period plus loans from the banks in proportion to the profits. This allows a primitive treatment of the role of banks' rules in the evolutionary process (see Nelson and Winter 1982: 291).

(13) The firm's desired investment is determined by the unit costs in the next period, a mark-up factor (the amount by which the price is increased before it is sold) influenced by the market share of the firm, and the rate of depreciation.

(14) The actual investment is the minimum of (11) and (12) provided that the result is not negative. The changes in physical capital influences production in the next period.

#### 2.2.6. State variables time $t + 1$

(15) The investment process has no time-lags. The adjusted physical capital stock is available to the industry's firms in period  $t + 1$ . By multiplying the capital stock  $K(it)$  with the new level of productivity  $A(it)$ , we have the production capacity of the firms of the industry in period  $t + 1$ . Similarly, the new productivity is available throughout the innovating or imitating firm.

## 2.1 NW77 in SWARM: results and analysis

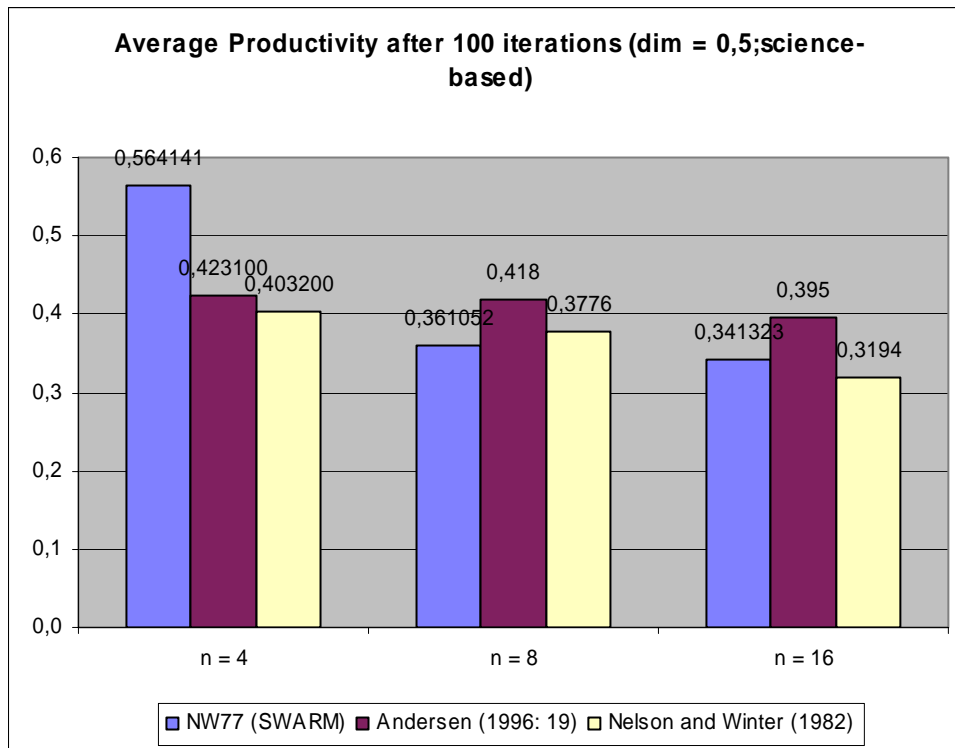
### 2.1.1 Initialisation of the parameters and source code

Half of the population of firms is set to be innovative (with  $r_{in} \geq 0$ ) the other half is set to be imitative (with  $r_{in} = 0$ ) and we are in the "science-based" case. I use exactly the same initial parameters as Nelson and Winter (1982) and Andersen (1996). All simulations stop after 100 iterations. One iteration represents 3 months for a firm. Thus, 100 iterations give a total time of 25 years. The initial parameters are:  $A_{init} = 0.16$ ;  $b = 1.0$ ;  $C = 0.16$ ;  $\delta = 0.03$ ;  $d_{in} = 0.125$ ;  $d_{im} = 67$ ;  $\eta = 1$ ;  $\phi = 0.01$ ;  $\psi = 1$ ;  $Trim = 0.4$ ;  $TR_{in} = 4.0$ ;  $\sigma = 0.05$ . The "easy imitation" case corresponds to  $d_{im} = 1.25$ ; the "difficult imitation" case corresponds to  $d_{im} = 0.5$ . The reader should look in Andersen (1996) for more details on the signification of each of these parameters and the reason why they are set with these initial values in the NW77 model. The location of the source code is given in Appendix A.

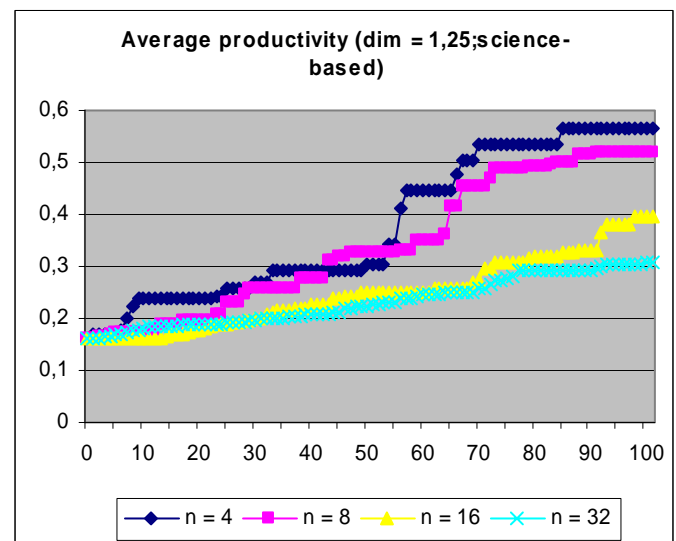
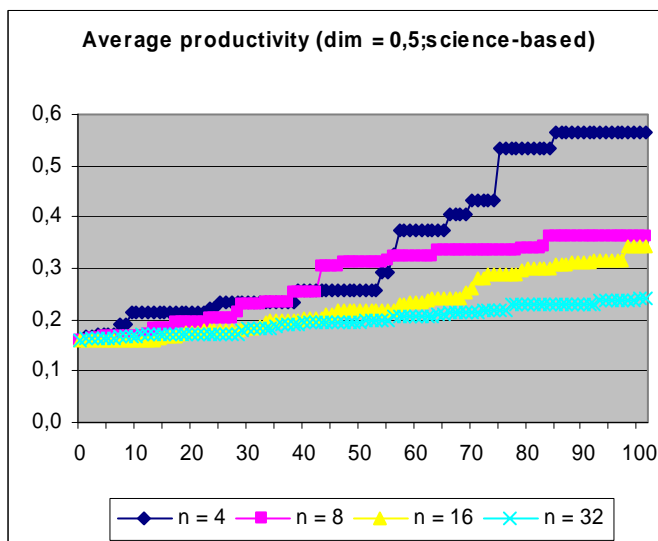
### 2.1.2 Industry level results

This section compares the results of NW77 implemented in SWARM with Andersen (1996: 19) and Nelson and Winter (1982: 275-307). Figure 2 below, compares the results obtained with different implementations of NW77 with different number of firms ( $n$ ). As we can see, we obtain similar results with 8 and 16 firms, but not exactly the same results due to the fact that we use different simulation environments and different random number generators for the Poisson and Normal distributions involved in the determination of innovative and imitative successes. However, the SWARM version with 4 firms shows a different average productivity, the reason might be the small number of firms.

**Figure 2. Comparison of the average productivity obtained with different implementations of NW77**



The figure bellow on the left, shows the average productivity under difficult imitation or high appropriability regime for different number of firms (n). In this industry regime, the four firms' case is characterised by higher jumps of average productivity than in the other cases, due to the small number of innovators (2) and imitators (2). The figure below on the right, presents the average productivity under easy imitation. We observe that the 8, 16 and 32 firms' case follows a different pattern than under difficult imitation and present a higher average productivity.

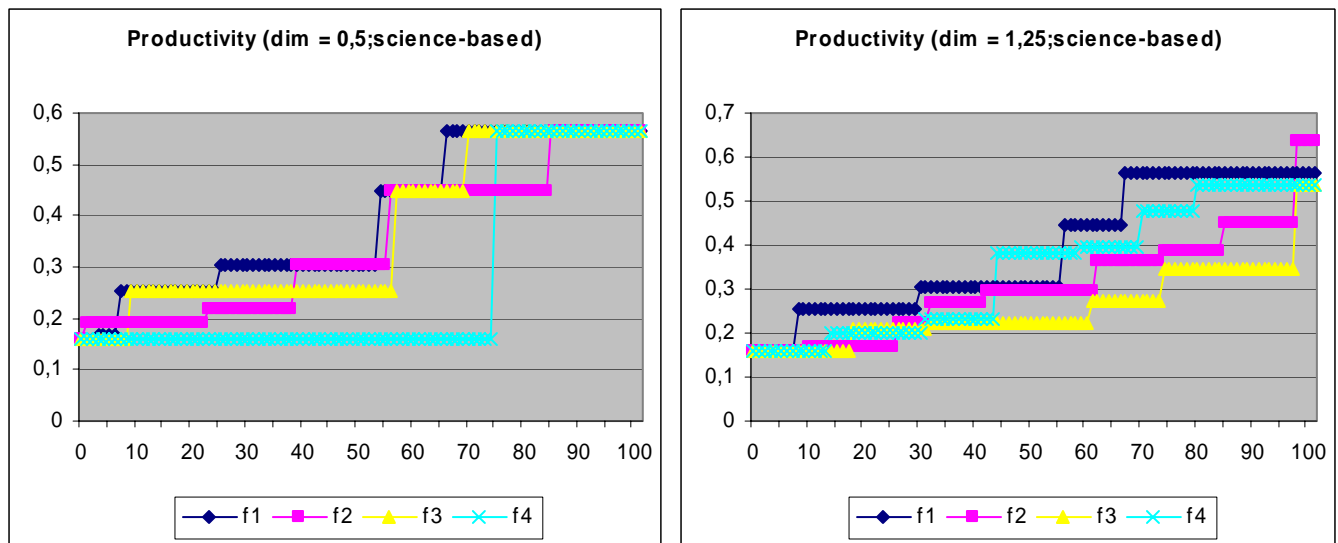


The reason why this pattern is different, which can only be found at the firm level, is that in the case corresponding to easy imitation, there is a higher probability that a weak firm may suddenly get access to the maximum productivity. Therefore, this pattern shows that the overall productivity level of the industry in Schumpeterian competition develops as a result of innovation and imitation by firms. The next section will take a closer look at the origin of this process at the firm level.

### 2.1.3 Firm level results

The figure below on the left shows the individual productivities for each firm under difficult imitation or high appropriability regime with four firms. The figure below on the right presents the individual productivities for each firm under easy imitation or low appropriability regime with four firms.

We can clearly see distinct pattern. On the left, the imitators (f3&4) only reach the maximum productivity after several iterations due to the difficult but not impossible access to the maximum productivity. On the right, the imitators (f3&f4) are already accessing the highest productivity after a couple of iterations and (f4) even manage to perform better than innovators (f2).



This difference between difficult and easy imitation respectively high and low appropriability regimes at the firm level explains the observed pattern at the industry level. Therefore, as explained previously, the presence of successful imitators affects the overall industry performance and explains the difference in average productivity observed at the industry level between the two regimes.

In the next section, I will relax the assumption that imitators can directly access the highest productivity and observe the effects of this modification, and if these effects find a possible explanation in evolutionary economics.

### 3 NW77 with local imitation: NWLocIm

In this model I modify NW77 to limit the scope of imitators. I put all firms on a torus surface and replace the imitator's global search for maximum productivity with a local search inside a Moore neighbourhood with eight neighbours. Thus I have modified NW77 rule (9), i.e., an imitative 'draw' or success means that the firm gets access to the best-practice technique and thus the highest productivity level obtained by any firm in the period. With, an imitative 'draw' or success means that the firm gets access to the local best-practice technique and thus the highest productivity level obtained by neighbour firms in the period. The location of the source code is given in Appendix A.

#### 3.1 Initialisation of the parameters

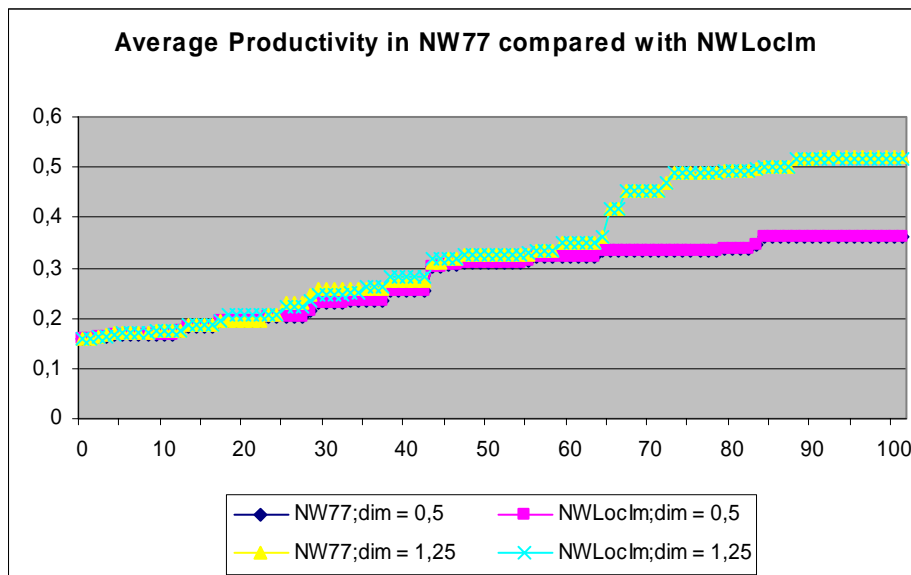
The modification does not concern the parameters and thus, I use the same initial parameters as in NW77.

#### 3.2 Industry level results

It is important to note that in the four firms' case, NWLocIm give the same results as NW77. This is because the Moore neighbourhood include all firms and thus is similar to the case with global access to the highest productivity. Therefore we will observe and analyse the results with NWLocIm with 8 or more firms.

Figure 3 below, shows that the average productivities for the 8 firms' case in NWLocIm are almost the same in NW77 for respectively easy imitation ( $\text{dim} = 1,25$ ) and difficult imitation ( $\text{dim} = 0,5$ ). Under difficult imitation, the results are the same because the change only concerns the imitation horizon and imitation is set to difficult. However, there is a very small difference under easy imitation, which is not possible to observe at this aggregated level and need to be investigated at the firm level.

Figure 3. Comparison between NW77 and NWLocIm



### 3.3 Firm level results

Figure 4 below, shows the results of NWLocIm and NW77 after 100 iterations with 8 firms under easy imitation. The only difference is very small and concerns the individual productivity of firm f8, an imitator, which is lower in NWLocIm than in NW77. It seems difficult to find an easy explanation by looking only at the individual productivity; therefore, I decide to look at individual market shares (s).

**Figure 4. Comparison between NWLocIm and NW77**

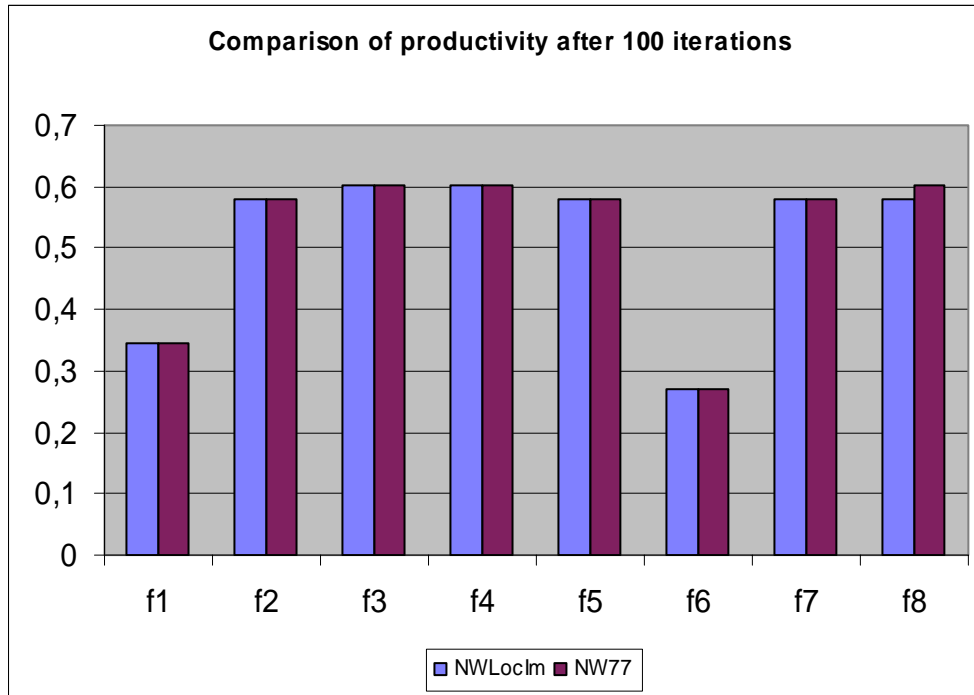
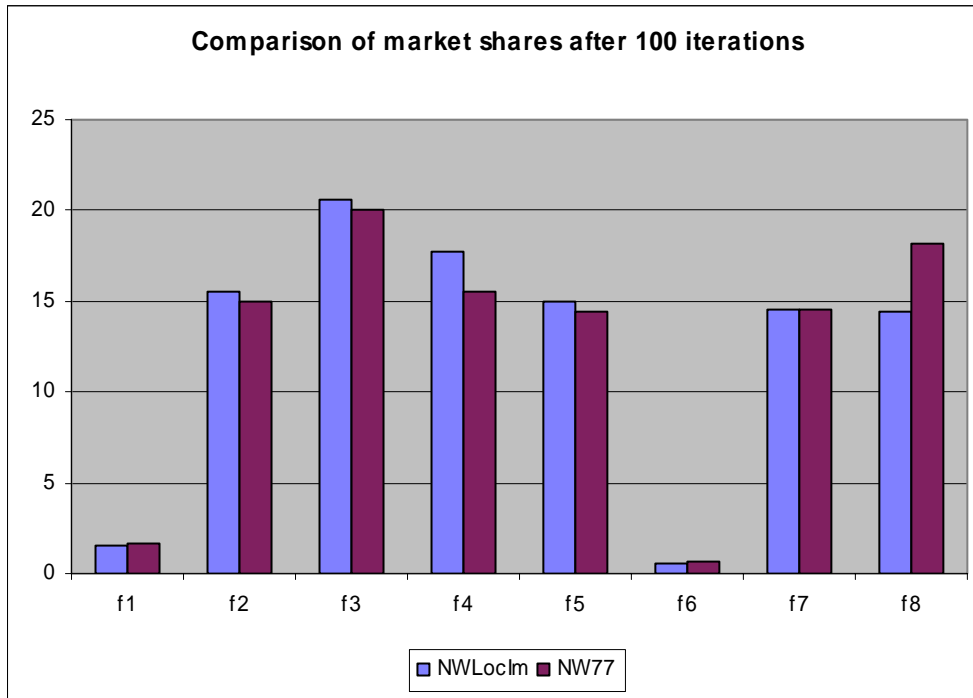


Figure 5 below, shows the results of NWLocIm and NW77 after 100 iterations with 8 firms under easy imitation. There are several differences concerning the individual market shares of innovators and imitators in NWLocIm compared with NW77. In NWLocIm, the imitators f5..f8 account for a lower total of market shares (44,56%) but the innovators f1..f4 account for a higher total of market shares (55,43%) compared with NW77, in which they have respectively 47,88% and 52,11%. But why are market shares different and can it explain the small difference observed at the industry level under easy imitation? An answer can be found with the analysis of another indicator (D), which measures the rate at which imitators can reach the maximum productivity available in the industry (Jonard and Yildizoglu 1998: 45).



**Figure 5. Comparison between NWLocIm and NW77**



If  $\underline{A}$  is the maximum productivity in the industry after 100 iterations,  $D = (1 / \underline{A}) * (\sum(A(i,j).K(i,j)) / \sum(K(i,j)))$ . I found  $D(\text{NWLocIm}) < D(\text{NW77})$ . The rate at which imitators reach the maximum productivity is lower in NWLocIm than in NW77, therefore, the slower diffusion of the best technology (found by successful innovators) seems to explain the difference of market shares.

In conclusion, the reason why market shares are different is that localised imitation, compared with global imitation, means limited scope of imitation for imitators and slower diffusion of the best technology of innovators. Slower diffusion of technology means lower market shares for imitators and consequently higher market shares for innovators.

## 4 Conclusion

We have seen that the NW77 model developed in Nelson and Winter (1982) and reconstructed in Andersen (1996) is very important for understanding the implications of Schumpeterian competition for industry structure and evolution. The adaptation of this model in the agent-based framework SWARM and its modification is only one small exploration of the large set of research opportunities offered by NW77. In this attempt to find evolutionary explanations corresponding to the modification I brought to NW77, I came across the work of many researchers in that field and I am sure that they have already made this change to the original model. However, to my knowledge nobody tried to adapt and modify NW77 in an agent-based environment like SWARM. The next step in the exploration of the opened research opportunities is to include more than two types of firms. For example, in addition to innovator and imitator firms, one could also have defensive firms. Defensive firms are in the first iterations identical to innovators; however, once they have an innovative success, they are essentially resting on their laurels and consolidating an established position (Freeman and Soete 1999: 273).

## ***References***

Andersen, E.S. (1996) The Nelson and Winter Models Revisited: Prototypes for Computer-Based Reconstruction of Schumpeterian Competition. DRUID Working Paper No. 96-2.

Freeman, C. and Soete L. (1999). The Economics of Industrial Innovation, Third Edition. MIT Press.

Jonard, N. and Yildizoglu, M. (1998). Technological diversity in an evolutionary industry model with localized learning and network externalities. *Structural and Economic Dynamics* 9: 35-53.

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## **APPENDIX A**

The source code of the SWARM version of NW77 and NWLocIm can be found at:  
<http://www.innovation.lth.se/files/fabrice/SwarmFest2005.zip>