## MEASURES OF TIPPING POINTS, ROBUSTNESS, AND PATH DEPENDENCE

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**Abstract:** This paper draws distinctions among various concepts sometimes collectively referred to as "tipping points" and relates them, where appropriate, to path dependence, robustness, and other properties of system dynamics. Complex systems thinking shifts the analytical focus to system dynamics and explaining processes, but we are still largely incapable of capturing system dynamics and measuring dynamical properties using quantitative measures. In response to this need I present a technique to represent system dynamics utilizing a specifically constructed Markov model and measure properties using various structural features, patterns, and "flows" on the corresponding network.

First I provide formal definitions of the foundational concepts required (attractor, reach, perimeter, etc.) in terms of the state transition diagram used. Then I provide definitions for multiple tipping point-related concepts and ways to measure their magnitudes using the foundational system dynamics terms. The next section builds upon the tipping point definitions to formally define and create measures for robustness, resilience, sustainability and similar concepts. Finally the technique is expanded to reveal path sensitivities: path preclusion, forcing, and path dependence. To facilitate applying these techniques to systems I also outline a general approach to generate the state transition diagram (even for continuous state systems) from data set and computational models.

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